Chapter 4: Factoring Algebraic Expressions

Getting Started, page 194

1. a) i), since there are two terms.
b) iv), since the number in front of x^2 is highlighted.
c) v), since 24 is broken up into its prime factors.
d) vi), since there is one term.
e) vii), since a variable is a symbol.
f) iii), since there are three factors.
g) ii), since the 4 is distributed and the brackets are taken away.
h) viii), Since both terms contain only the variable y.

2. a) $4x - 6y + 8y - 5x = 4x - 5x + 8y - 6y$
$= -x + 2y$

b) $5ab - 6a^2 + 6ab - 3a^2 - 11ab + 9b^2$
$= 5ab + 6ab - 11ab - 6a^2 - 3a^2 + 9b^2$
$= -9a^2 + 9b^2$

c) $(2x - 5y) + (7x + 4) - (5x - y)$
$= 2x - 5y + 7x + 4 - 5x + y$
$= 2x + 7x - 5x + y - 5y + 4$
$= 4x - 4y + 4$

d) $(7a - 2ab) - (4b + 5a) + (ab - 3a)$
$= 7a - 2ab - 4b - 5a + ab - 3a$
$= 7a - 3a - 5a + ab - 2ab - 4b$
$= -a - ab - 4b$

3. a) $7(2x - 5) = 14x - 35$

b) $-5x(3x^2 - 4x + 5)$
$= -15x^3 + 20x^2 - 25x$

c) $2(4x^2 + 3x + 1) - 2x(8x - 3)$
$= 8x^2 + 6x + 2 - 16x^2 + 6x$
$= 8x^2 - 16x^2 + 6x + 6x + 2$
$= -8x^2 + 12x + 2$

d) $(d - 6)(d + 2) = d(d + 2) - 6(d + 2)$
$= d^2 + 2d - 6d - 12$
$= d^2 - 4d - 12$

e) $(3a - 7b)(4a - 3b)$
$= 3a(4a - 3b) - 7b(4a - 3b)$
$= 12a^2 - 9ab - 28ab + 21b^2$
$= 12a^2 - 37ab + 21b^2$

f) $6x(2x + 1)^2 = 6x(2x + 1)(2x + 1)$
$= 6x[2x(2x + 1) + 1(2x + 1)]$
$= 6x(4x^2 + 2x + 2x + 1)$
$= 6x(4x^2 + 4x + 1)$
$= 24x^3 + 24x^2 + 6x$

4. The expression and product can be determined from the algebra tiles.
a) $4(2x + 5) = 8x + 20$
b) $2x(2x + 2) = 4x^2 + 4x$

c) $(x + 2)(3x + 2) = x(3x + 2) + 2(3x + 2)$
$= 3x^2 + 2x + 6x + 4$
$= 3x^2 + 8x + 4$

d) $(x - 2)(3x + 2) = x(3x + 2) - 2(3x + 2)$
$= 3x^2 + 2x - 6x - 4$
$= 3x^2 - 4x - 4$

5. a) $(x^5)(x^7) = x^{5+7}$
$= x^{12}$

b) $(-6a^2)(3a^4)(2a) = (-6)(3)(2)\,a^{2+4+1}$
$= -36a^7$

c) $(4y)(3y^2) \div 2y^3 = 4(3)y^{2+1} \div 2y^3$
$= 12y^3 \div 2y^3$
$= 6$

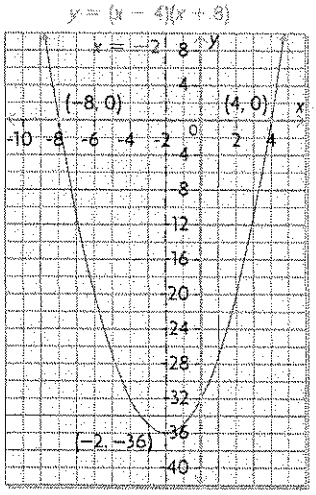
d) $20z^5 \div (-4z^3)(-z^2) = (-5z^2)(-z^2)$
$= 5z^4$

6. Answers may vary, e.g.,
a) $5x^3$ **b)** $4a + 6b$ **c)** $2x + 3y + 4$ **d)** $x^2 - 8x + 9$

7. a) The factors of 28 are 1, 2, 4, 7, 14, and 28. The factors of 35 are 1, 5, 7, and 35. The greatest common factor is 7.
b) The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. The factors of 63 are 1, 3, 7, 9, 21, and 63. The greatest common factor is 9.
c) The factors of 99 are 1, 3, 9, 11, 33, and 99. The factors of 90 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, and 90. The greatest common factor is 9.
d) The factors of $4x$ are 1, 2, 4, and x. The factors of 8 are 1, 2, 4, and 8. The greatest common factor is 4.
e) The factors of 25 are 1, 5 and 25. The factors of $5x^2$ are 1, 5, x, and x^2. The greatest common factor is 5.
f) The factors of $12y$ are 1, 2, 3, 4, 6, 12, and y. The factors of $6x$ are 1, 2, 3, 6, and x. The greatest common factor is 6.

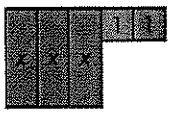
8. a) To determine the x-intercepts, set each factor equal to zero.
$x - 4 = 0$
$x = 4$
$x + 8 = 0$
$x = -8$
The x-intercepts are 4 and –8.
The axis of symmetry is midway between the two zeros
$x = 0.5(4 - 8)$
$= 0.5(-4)$
$= -2$
The equation of the axis of symmetry is $x = -2$.
When $x = -2$,
$y = (x - 4)(x + 8)$
$y = (-2 - 4)(-2 + 8)$
$y = (-6)(6)$
$y = -36$
The vertex is (–2, –36).

b)



c) $y = (x - 4)(x + 8)$
$y = x(x + 8) - 4(x + 8)$
$y = x^2 + 8x - 4x - 32$
$y = x^2 + 4x - 32$

9. a)



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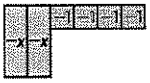

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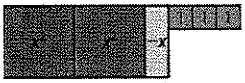
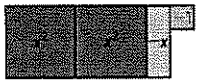


d)

e)



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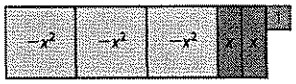
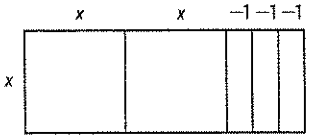


10. a) iii) **b)** ii) **c)** i)

11. a)



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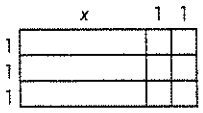

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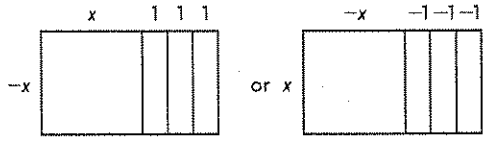

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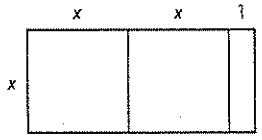


12. Answers may vary, e.g.,
a) Agree. Since all even numbers are divisible by 2 and 2 is a prime number.
b) Disagree; $6x^2y^3$ cannot be factored.
Agree. $6x^2y^3$ can be written many ways such as $(6x^2)(y^3)$, $(2x)(3xy^3)$, $(3x)(y)(2xy^2)$, and $(2y^3)(3x)(x)$.
c) Disagree. There are many other ways to factor 100, such as 2×50, 4×25, 5×20, and $2 \times 2 \times 5 \times 5$.

Lesson 4.1; Common Factors in Polynomials, page 202

1. a) i) The expression is $4x^2 - 12x$. There are two rows of x thus $2x$ is a common factor.
ii) The greatest common factor between 4 and 12 is 4, and the greatest common factor between x^2 and x is x.
The greatest common factor between $4x^2$ and $-12x$ is $4x$.
b) i) The expression is $-6x^2 + 12x$. There are three rows of x thus $3x$ is a common factor.
ii) The greatest common factor between 6 and 12 is 6, and the greatest common factor between x^2 and x is x. Thus the greatest common factor between $-6x^2$ and $12x$ is $6x$.
2. a) i) The expression is $4x + 16$. There are four rows of 1 so 4 is a common factor.
ii) Other factors include -4, 2, -2, 1, and -1, all of which are factors of 4.
b) i) The expression is $6x^2 + 8x$. There are two rows of x thus $2x$ is a common factor.
ii) Other factors include $-2x$, x, $-x$, 2, -2, 1, and -1, all of which are factors of $2x$.
3. a) factors of 6: 1, 2, 3, 6; factors of 10: 1, 2, 5, 10;
The greatest common factor (GCF) of 6 and 10 is 2. The GCF of x and x is x. The GCF of $6x$ and $10x$ is $2x$.
b) factors of 15: 1, 3, 5, 15; factors of 20: 1, 2, 4, 5, 10, 20;
The GCF of 15 and 20 is 5 and the GCF of a^3 and a^2 is a^2.
The GCF of $15a^3$ and $20a^2$ is $5a^2$.
c) The GCF of a and a^2 is a. The GCF of b and b^2 is b. The GCF of ab and a^2b^2 is ab.
d) factors of 2: 1, 2; factors of 8: 1, 2, 4, 8;
The GCF of the coefficients is 2, the GCF of the y terms is y^4, and the GCF of the x terms is x^3. The GCF of $-2x^4y^4$ and $8x^3y^5$ is $2x^3y^4$.
4. a) Since only a coefficient is common in all three terms then the GCF must be a coefficient.
factors of 3: 1, 3; factors of 9: 1, 3, 9;factors of 12: 1, 2, 3, 4, 6, 12; The GCF is 3.
b) Since the two coefficients are both distinct prime numbers the GCF of the coefficients is 1. Both terms also contain x terms, their GCF is x. Thus the GCF is x.
c) The GCF of x and x^2 is x and y and y^2 is y.
The GCF is xy.
d) Since 3 and 4 have a GCF of 1, and $(x - 1)$ is multiplied by each term then the GCF for this expression is $x - 1$.
5. a) For the left side to equal the right side the coefficient in front of xy must be 8. $2 \times ? = 8$. The missing factor is 4.
b) For the left side to equal the right side the coefficient in front of x^2 must be -6. $3 \times ? = -6$. The missing factor is -2.
c) $\blacksquare 5x^2 = 15x^4z$

$$\blacksquare = \frac{15x^4z}{5x^2}$$

$$\blacksquare = 3x^{4-2}z$$

$$\blacksquare = 3x^2z$$

The missing factor is $3x^2z$.

d) $\blacksquare 7b^3 = -49a^2b^5$

$$\blacksquare = \frac{-49a^2b^5}{7b^3}$$

$$\blacksquare = -7a^2b^{5-3}$$

$$\blacksquare = -7a^2b^2$$

The missing factor is $-7a^2b^2$.

e) $\blacksquare 3y^3 = -12x^3y^3$

$$\blacksquare = \frac{-12x^3y^3}{3y^3}$$

$$\blacksquare = -4x^3$$

The missing factor is $-4x^3$.

f) $\blacksquare(-5m^2n) = 30m^2n^3$

$$\blacksquare = \frac{30m^2n^3}{-5m^2n}$$

$$\blacksquare = -6n^2$$

The missing factor is $-6n^2$.

6. a) To get $4x$, x must be multiplied by 4. If 4 is input as the missing factor then

$4(x - y) = 4(x) - 4(y)$

$= 4x - 4y$

The missing factor is 4.

b) $2\blacksquare = 8x - 2y$

$$\blacksquare = \frac{8x}{2} - \frac{2y}{2}$$

$$\blacksquare = 4x - y$$

The missing factor is $4x - y$.

c) To get $5a$, a must be multiplied by 5. If 5 is input as the missing factor then

$5(a + 2b) = 5(a) + 5(2b)$

$= 5a + 10b$

The missing factor is 5.

d) $4\blacksquare = 36x^2 - 32y^3$

$$\blacksquare = \frac{36x^2 - 32y^3}{4}$$

$$\blacksquare = \frac{36x^2}{4} - \frac{32y^3}{4}$$

$$\blacksquare = 9x^2 - 8y^3$$

The missing factor is $9x^2 - 8y^3$

e) To get $-6y$, y must be multiplied by -6. If -6 is input as the missing factor then

$-6(4x^2 + y) = -6(4x^2) - 6(y)$

$= -24x^2 - 6y$

The missing factor is -6.

f) $9a^3\blacksquare = 45a^4 - 54a^3$

$$\blacksquare = \frac{45a^4}{9a^3} - \frac{54a^3}{9a^3}$$

$$\blacksquare = 5a - 6$$

The missing factor is $5a - 6$.

7. a) Since only a coefficient is common in all three terms then the GCF must be a coefficient.
factors of 7: 1, 7; factors of 14: 1, 7, 2, 14;
factors of 21: 1, 3, 7, 21;
The greatest common factor is 7.

b) factors of 3: 1, 3; factors of 15: 1, 3, 5, 15;
The GCF of the coefficients is 3 and the GCF the variables is b. The greatest common factor of the expression is $3b$.

c) Since only a coefficient is common in all three terms then the GCF must be a coefficient.
factors of 12: 1, 2, 3, 4, 6, 12; factors of 8: 1, 2, 4, 8;
factors of 16: 1, 2, 4, 8, 16;
The greatest common factor is 4.

d) factors of -25: ± 1, ± 5, ± 25; factors of -10: ± 1, ± 2, ± 5, ± 10;The GCF of the coefficients is ± 5 and the GCF of the variables is m. The GCF is $5m$ or $-5m$.

e) factors of 3: 1, 3; factors of 9: 1, 3, 9;
factors of 15: 1, 3, 5, 15;
The GCF of the coefficients is 3 and the GCF the variables is d^2. The greatest common factor of the expression is $3d^2$.

f) There are only variables in this expression, so the GCF will be the lowest power of y. The GCF is y^2.

8. Answers may vary, e.g.,

a) All the coefficients have the greatest common factor of 3. I placed 3 outside the brackets and divided the expression by 3.

$9x^2 - 6x + 18 = 3(3x^2 - 2x + 6)$

b) Each term in this expression is a multiple of 5 and contains a common factor of a. I divided each term by $5a$ and placed $5a$ outside the bracket.

$25a^2 - 20a = 5a(5a - 4)$

c) The greatest number of 27 and 9 is 9. I determined that the greatest common factor for y^3 and y^4 is y^3. I determined that the GCF is $9y^3$, and then factored out $9y^3$.

$27y^3 - 9y^4 = 9y^3(3 - y)$

d) Both $2b$ and 5 are multiplied by $b + 4$, so $b + 4$ is a common factor. I wrote an equivalent expression using the distributive property.

$2b(b + 4) + 5(b + 4) = (b + 4)(2b + 5)$

e) Both $4c$ and 5 are multiplied by $c - 3$ so $c - 3$ is a common factor. I wrote an equivalent expression using the distributive property.

$4c(c - 3) - 5(c - 3) = (c - 3)(4c - 5)$

f) Both x and $x + 1$ are multiplied by $3x - 5$ so $3x - 5$ is a common factor. I wrote an equivalent expression using the distributive property.

$x(3x - 5) + (3x - 5)(x + 1) = (3x - 5)(2x + 1)$

9. a) There is a d common to each term.

$dc^2 - 2acd + 3da^2 = d(c^2 - 2ac + 3a^2)$

b) There is a 5, c, and a common to each term. When factored, the expression is $5ac(-2a + 4 - c^2)$.

c) Since the last term is a constant only a number can be factored out. Since each term is a multiple of 5, 5 will be factored out. $10ac^2 - 15a^2c + 25 = 5(2ac^2 - 3a^2c + 5)$.

d) The polynomial has a common factor of $2a^2c^2$. I divided each term by the common factor.

$$2a^2c^4 - 4a^3c^3 + 6a^4c^2 = 2a^2c^2\left(\frac{2a^2c^4}{2a^2c^2} - \frac{4a^3c^3}{2a^2c^2} + \frac{6a^4c^2}{2a^2c^2}\right)$$
$$= 2a^2c^2(c^2 - 2ac + 3a^2)$$

e) The polynomial has the common factor ac. I divided each term by the common factor and then wrote an equivalent expression using the distributive property.
$3a^5c^3 - 2ac^2 + 7ac = ac(3a^4c^2 - 2c + 7)$

f) The polynomial has the common factor $2cd$. I divided each term by the common factor and then wrote an equivalent expression using the distributive property.
$10c^3d - 8cd^2 + 2cd = 2cd(5c^2 - 4d + 1)$

Polynomials in parts a) and d) have the same trinomial factor.

10. a) $ax - ay + bx - by = a(x - y) + b(x - y)$
$= (x - y)(a + b)$

b) $10x^2 + 5x - 6xy - 3y = 5x(2x + 1) - 3y(2x + 1)$
$= (2x + 1)(5x - 3y)$

c) $3mx + 3my + 2x + 2y = 3m(x + y) + 2(x + y)$
$= (x + y)(3m + 2)$

d) $5my + tm + 5ny + tn = m(5y + t) + n(5y + t)$
$= (5y + t)(m + n)$

e) $5wx - 10w - 3tx + 6t = 5w(x - 2) - 3t(x - 2)$
$= (x - 2)(5w - 3t)$

f) $4mnt - 16mn - t + 4 = 4mn(t - 4) - 1(t - 4)$
$= (t - 4)(4mn - 1)$

11. The following formula contains all the information given about the length of the sides.

$$A = \frac{(b_1 + b_2)h}{2}$$
$$A = \frac{(4b + 1 + 3b)h}{2}$$
$$A = \frac{(7b + 1)h}{2}$$

Also, $A = 70b + 10$. Equate these two equations to solve for h.

$$70b + 10 = \frac{(7b + 1)h}{2}$$
$$140b + 20 = (7b + 1)h$$
$$h = \frac{140b + 20}{7b + 1}$$
$$h = 20$$

The height of the trapezoid is 20 units.

12. a) i) $y = 2x^2 - 10x$

$$y = 2x\left(\frac{2x^2}{2x} - \frac{10x}{2x}\right)$$
$$y = 2x(x - 5)$$

The equation in factored form is $y = 2x(x - 5)$.

ii) To determine the zeros, set each factor equal to zero.
$2x = 0$
$x = 0$;
$x - 5 = 0$
$x = 5$

The zeros are 0 and 5.
The axis of symmetry is midway between the two zeros.

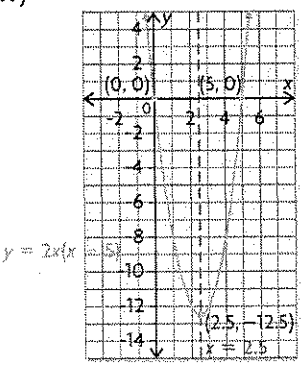
$$x = \frac{5 + 0}{2}$$
$$x = 2.5$$

The equation of the axis of symmetry is $x = 2.5$.

iii) When $x = 2.5$ then
$y = 2x^2 - 10x$
$y = 2(2.5)^2 - 10(2.5)$
$y = 12.5 - 25$
$y = -12.5$

The vertex is (2.5, –12.5).

iv)



b) i) $y = -x^2 - 8x$

$$y = -x\left(\frac{-x^2}{-x} - \frac{8x}{-x}\right)$$
$$y = -x(x + 8)$$

The equation in factored form is $y = -x(x + 8)$.

ii) Determine the zeros by setting each factor equal to zero.
$-x = 0$;
$x + 8 = 0$
$x = -8$

The zeros are 0 and 8.
The axis of symmetry is midway between these two points.

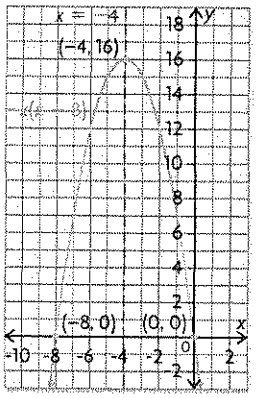
$$x = \frac{-8 + 0}{2}$$
$$x = -4$$

The equation of the axis of symmetry is $x = -4$.

iii) When $x = -4$ then
$y = -x^2 - 8x$
$y = -(-4)^2 - 8(-4)$
$y = -16 + 32$
$y = 16$

The vertex is (–4, 16).

iv)



13. Let l represent length, w represent width and h represent height.

$A = lw + lw + wh + wh + lh + lh$
$A = 2lw + 2wh + 2lh$
$A = 2(lw + wh + lh)$

A factored formula for the area of a rectangular prism is $A = 2(lw + wh + lh)$.

14. Begin by determining the maximum value of the first equation, $y = 10x - x^2$. The zeros are:

$y = x(10 - x)$
$x = 0$ and $x = 10$

The axis of symmetry is:

$$x = \frac{0+10}{2}$$
$$x = 5$$

The maximum value is,

$y = 10(5) - (5)^2$
$y = 50 - 25$
$y = 25$

Now determine the minimum value of the second equation,

$y = 3x^2 - 30x$.

The zeros are:

$y = 3x(x - 10)$
$x = 0$ or $x = 10$

The equation of the axis of symmetry is:

$$x = \frac{0+10}{2}$$
$$x = 5$$

The maximum value is,

$y = 3(5)^2 - 30(5)$
$y = 75 - 150$
$y = -75$

Since one value is 25 and the other is –75, there are 100 units between them. The distance between the maximum and minimum points is 100 units.

15. a) Answers may vary, e.g.,

$20x + 5x^2 = 5x(20x \div 5x + 5x^2 \div 5x)$
$= 5x(4 + x)$
$10x^2 - 5x = 5x(10x^2 \div 5x - 5x \div 5x)$
$= 5x(2x - 1)$
$5x^2 + 50xy = 5x(5x^2 \div 5x + 50xy \div 5x)$
$= 5x(x + 10y)$

b) Answers may vary, e.g.,

$3x^2 - 6x + 3xy = 3x(3x^2 \div 3x - 6x \div 3x + 3xy \div 3x)$
$= 3x(x - 2 + y)$
$9x^2 + 12x - 6xy = 3x(9x^2 \div 3x + 12x \div 3x - 6xy \div 3x)$
$= 3x(3x + 4 - 2y)$
$6x^2 + 3x - 3ax = 3x(6x^2 \div 3x + 3x \div 3x - 3ax \div 3x)$
$= 3x(2x + 1 - a)$

16. Both Marek and Jen are correct since $-5x^3 + 10x^2 - 20x$ can be factored either as $-5x(x^2 - 2x + 4)$ or as $5x(-x^2 + 2x - 4)$. Since both expressions are equivalent, $-5x$ and $5x$ are both acceptable greatest common factors.

17. Let x, $(x + 1)$, and $(x + 2)$ be the three consecutive integers.

$S = x^2 + (x + 1)^2 + (x + 2)^2 + 1$
$S = x^2 + (x^2 + 2x + 1) + (x^2 + 4x + 4) + 1$
$S = 3x^2 + 6x + 6$
$S = 3(x^2 + 2x + 2)$

3 is a common factor, so the result must be divisible by 3.

18. You can expand the factored expression using the distributive property. Expanding is the opposite of factoring, so if you expand your factors correctly and arrive at the same expression you started with, you will know that your factors are correct.

19. The area of a square is one side squared, thus the area of this square will be $(2r)^2 = 4r^2$. The area of this circle will be πr^2. To find the area of the shaded region subtract the area of the circle from the area of the square.

$A = 4r^2 - \pi r^2$
$A = r^2(4 - \pi)$

20. a) $\frac{2x^2y + 3xy^2}{xy} = \frac{xy(2x+3y)}{xy}$
$= 2x + 3y$

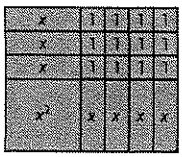
b) $\frac{6x^3y + 12x^3y^2}{6x^3y} = \frac{6x^3y(1+2y)}{6x^3y}$
$= 1 + 2y$

c) $\frac{-12x^3y^2 - 18x^2y^3}{6x^2y^2} = \frac{-6x^2y^2(2x+3y)}{6x^2y^2}$
$= -(2x + 3y)$

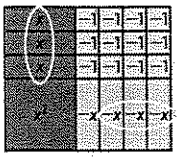
d) $\frac{3x^4 + 6x^3 + 9x^2}{3x^2} = \frac{3x^2(x^2+2x+3)}{3x^2}$
$= x^2 + 2x + 3$

Lesson 4.2: Exploring the Factorization of Trinomials, page 206

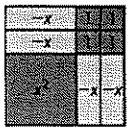
1. a) The factors are $(x + 3)$ and $(x + 4)$.



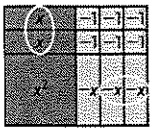
b) The factors are $(x + 3)$ and $(x - 4)$.
Equivalent added tiles are circled.



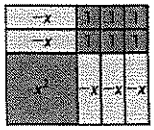
c) The factors are $(x - 2)$ and $(x - 2)$.



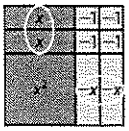
d) The factors are $(x + 2)$ and $(x - 3)$.
Equivalent added tiles are circled.



e) The factors are $(x - 2)$ and $(x - 3)$.



f) The factors are $(x + 2)$ and $(x - 2)$.
Equivalent added tiles are circled.



2. a) $(x + 3)(x + 4) = x(x + 4) + 3(x + 4)$
$= x^2 + 4x + 3x + 12$
$= x^2 + 7x + 12$

b) $(x + 3)(x - 4) = x(x - 4) + 3(x - 4)$
$= x^2 - 4x + 3x - 12$
$= x^2 - x - 12$

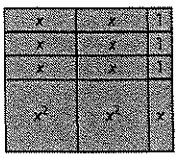
c) $(x - 2)(x - 2) = x(x - 2) - 2(x - 2)$
$= x^2 - 2x - 2x + 4$
$= x^2 - 4x + 4$

d) $(x + 2)(x - 3) = x(x - 3) + 2(x - 3)$
$= x^2 - 3x + 2x - 6$
$= x^2 - x - 6$

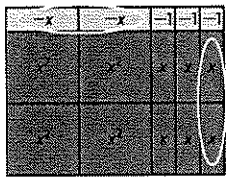
e) $(x - 2)(x - 3) = x(x - 3) - 2(x - 3)$
$= x^2 - 3x - 2x + 6$
$= x^2 - 5x + 6$

f) $(x + 2)(x - 2) = x(x - 2) + 2(x - 2)$
$= x^2 - 2x + 2x - 4$
$= x^2 - 4$

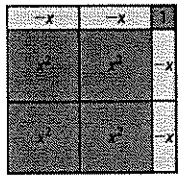
3. a) The factors are $(x + 3)$ and $(2x + 1)$.



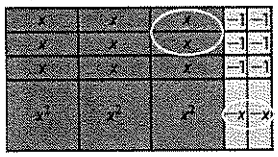
b) The factors are $(2x - 1)$ and $(2x + 3)$.
Equivalent added tiles are circled.



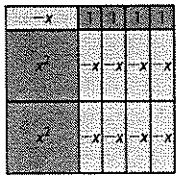
c) The factors are $(2x - 1)$ and $(2x - 1)$.



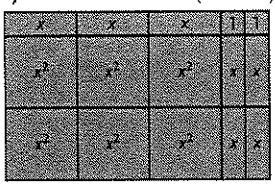
d) The factors are $(x + 3)$ and $(3x - 2)$.
Equivalent added tiles are circled.



e) The factors are $(2x - 1)$ and $(x - 4)$.



f) The factors are $(2x + 1)$ and $(3x + 2)$.



4. a) $(x + 3)(2x + 1) = x(2x + 1) + 3(2x + 1)$
$= 2x^2 + x + 6x + 3$
$= 2x^2 + 7x + 3$

b) $(2x - 1)(2x + 3) = 2x(2x + 3) - 1(2x + 3)$
$= 4x^2 + 6x - 2x - 3$
$= 4x^2 + 4x - 3$

c) $(2x - 1)(2x - 1) = 2x(2x - 1) - 1(2x - 1)$
$= 4x^2 - 2x - 2x + 1$
$= 4x^2 - 4x + 1$

d) $(x + 3)(3x - 2) = x(3x - 2) + 3(3x - 2)$
$= 3x^2 - 2x + 9x - 6$
$= 3x^2 + 7x - 6$

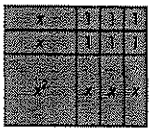
e) $(2x - 1)(x - 4) = 2x(x - 4) - 1(x - 4)$
$= 2x^2 - 8x - x + 4$
$= 2x^2 - 9x + 4$

f) $(2x + 1)(3x + 2) = 2x(3x + 2) + 1(3x + 2)$
$= 6x^2 + 4x + 3x + 2$
$= 6x^2 + 7x + 2$

Lesson 4.3; Factoring Quadratics: $x^2 + bx + c$, page 211

1. a) The algebra tiles represent $x^2 + 5x + 6$.

b)



c) The factors are $(x + 3)$ and $(x + 2)$.

2. a) By counting the tiles the expression is $x^2 - x - 6$. Its factors are $(x - 3)$ and $(x + 2)$.

b) By counting the tiles the expression is $x^2 - 7x + 12$. Its factors are $(x - 4)$ and $(x - 3)$.

3. a) I need two numbers whose product is 21. One is -7 so the other must be -3. The missing factor is $(x - 3)$. To check, $-7 - 3 = -10$, the coefficient of x.

b) I need two numbers whose product is -32. One is -4 so the other must be 8. The missing factor is $(x + 8)$. To check, $-4 + 8 = +4$, the coefficient of x.

c) I need two numbers whose product is -63. One is 7 so the other must be -9. The missing factor is $(x - 9)$. To check, $7 - 9 = -2$, the coefficient of x.

d) I need two numbers whose product is 45. One is 9 so the other must be 5. The missing factor is $(x + 5)$. To check, $9 + 5 = 14$, the coefficient of x.

4. a) Two numbers whose product is 1 and whose sum is 2 are 1 and 1. The factored expression is $(x + 1)(x + 1)$.

b) Two numbers whose product is 1 and whose sum is -2 are -1 and -1. The factored expression is $(x - 1)(x - 1)$.

c) Two numbers whose product is -3 and whose sum is -2 are -3 and 1. The factored expression is $(x - 3)(x + 1)$.

d) Two numbers whose product is 9 and whose sum is 6 are 3 and 3. The factored expression is $(x + 3)(x + 3)$.

e) Two numbers whose product is 4 and whose sum is -4 are -2 and -2. The factored expression is $(x - 2)(x - 2)$.

f) Two numbers whose product is -12 and whose sum is -4 are -6 and 2. The factored expression is $(x - 6)(x + 2)$.

5. a) By counting the algebra tiles the expression is $x^2 + 5x - 2x - 10$, which equals $x^2 + 3x - 10$. Its factors are $(x + 5)$ and $(x - 2)$.

b) By counting the algebra tiles the expression is $x^2 - 4x + 4$. Its factors are $(x - 2)$ and $(x - 2)$.

6. a) I need two numbers whose product is 24. One is 3 so the other must be 8. The missing factor is $(x + 8)$. To check, $3 + 8 = 11$, the coefficient of x.

b) I need two numbers whose product is 56. One is -7 so the other must be -8. The missing factor is $(c - 8)$. To check, $-7 - 8 = -15$, the coefficient of c.

c) I need two numbers whose product is -60. One is -15 so the other must be 4. The missing factor is $(a + 4)$. To check, $-15 + 4 = -11$, the coefficient of a.

d) I need two numbers whose product is -44. One is 2 so the other must be -22. The missing factor is $(y - 22)$. To check, $2 - 22 = -20$, the coefficient of y.

e) I need two numbers whose product is -48. One is 8 so the other must be -6. The missing factor is $(b - 6)$. To check, $8 - 6 = 2$, the coefficient of b.

f) I need two numbers whose product is 90. One is -10 so the other must be -9. The missing factor is $(z - 9)$. To check, $-10 - 9 = -19$, the coefficient of z.

7. a) Two numbers whose product is 3 and whose sum is 4 are 1 and 3. The factored expression is $(x + 3)(x + 1)$.

b) Two numbers whose product is 20 and whose sum is -9 are -5 and -4. The factored expression is $(a - 5)(a - 4)$.

c) Two numbers whose product is -16 and whose sum is 8 are -4 and -4. The factored expression is $(m - 4)(m - 4)$.

d) Two numbers whose product is -6 and whose sum is 1 are 3 and -2. The factored expression is $(n + 3)(n - 2)$.

e) Two numbers whose product is -16 and whose sum is 6 are 8 and -2. The factored expression is $(x + 8)(x - 2)$.

f) Two numbers whose product is -16 and whose sum is 15 are 16 and -1. The factored expression is $(x + 16)(x - 1)$.

8. a) Two numbers whose product is 16 and whose sum is -10 are -8 and -2. The factored expression is $(x - 8)(x - 2)$.

b) Two numbers whose product is -40 and whose sum is 6 are 10 and -4. The factored expression is $(y + 10)(y - 4)$.

c) Two numbers whose product is -56 and whose sum is -1 are -8 and 7. The factored expression is $(a - 8)(a + 7)$.

d) Two numbers whose product is -14 and whose sum is -5 are -7 and 2. The factored expression is $(w - 7)(w + 2)$.

e) Two numbers whose product is 32 and whose sum is -12 are -8 and -4. The factored expression is $(m - 8)(m - 4)$.

f) Two numbers whose product is -42 and whose sum is 1 are 7 and -6. The factored expression is $(n + 7)(n - 6)$.

9. a) Begin by factoring out the coefficient in front of x^2. $3x^2 + 24x + 45 = 3(x^2 + 8x + 15)$ Now two numbers whose product is 15 and whose sum is 8 are 3 and 5. Thus $3x^2 + 24x + 45 = 3(x + 3)(x + 5)$.

b) Begin by factoring out the coefficient in front of y^2. $2y^2 - 2y - 60 = 2(y^2 - y - 30)$. Now two numbers whose product is -30 and whose sum is -1 are 5 and -6. Thus $2y^2 - 2y - 60 = 2(y + 5)(y - 6)$.

c) Begin by factoring out the coefficient in front of v^2. $3v^2 + 9v + 6 = 3(v^2 + 3v + 2)$. Now two numbers whose product is 2 and whose sum is 3 are 1 and 2. Thus $3v^2 + 9v + 6 = 3(v + 1)(v + 2)$.

d) Begin by factoring out the coefficient in front of n^2. $6n^2 + 24n - 30 = 6(n^2 + 4n - 5)$ Now two numbers whose product is -5 and whose sum is 4 are -1 and 5. Thus $6n^2 + 24n - 30 = 6(n - 1)(n + 5)$.

e) Since there is an x present in each term I can factor it out. $x^3 + 5x^2 + 4x = x(x^2 + 5x + 4)$. Now two numbers whose product is 4 and whose sum is 5 are 4 and 1. Thus $x^3 + 5x^2 + 4x = x(x + 4)(x + 1)$.
f) Since there is an $7x^2$ present in each term I can factor it out. $7x^4 + 28x^3 - 147x^2 = 7x^2(x^2 + 4x - 21)$
Now two numbers whose product is -21 and whose sum is 4 are 7 and -3.
Thus $7x^4 + 28x^3 - 147x^2 = 7x^2(x + 7)(x - 3)$.

10. Answers may vary, e.g.,

i) $(x - 2)(x + 1) = x(x + 1) - 2(x + 1)$
$= x^2 + x - 2x - 2$
$= x^2 - x - 2$

ii) $(x - 2)(x - 4) = x(x - 4) - 2(x - 4)$
$= x^2 - 4x - 2x + 8$
$= x^2 - 6x + 8$

iii) $(x - 2)(x - 2) = x(x - 2) - 2(x - 2)$
$= x^2 - 2x - 2x + 4$
$= x^2 - 4x + 4$

Three polynomials with $(x - 2)$ as a factor are $x^2 - x - 2$, $x^2 - 6x + 8$ and $x^2 - 4x + 4$.

11. Answers may vary, e.g., Martina factored $x^2 - 15x + 44$ as $(4 - x)(11 - x)$. We can see that both answers are correct by multiplying each factor by -1, which gives $(-4 + x)(-11 + x)$ or $(x - 4)(x - 11)$. Since we multiplied twice by -1, and since $(-1)(-1) = 1$, we have not changed the value of our answer.

12. a) Two numbers whose product is 15 and whose sum is 8 are 3 and 5. Thus,
$a^2 + 8a + 15 = (a + 3)(a + 5)$.
b) Since 3 is a factor of each term it can be factored out: $3x^2 - 21x - 54 = 3(x^2 - 7x - 18)$. Two numbers whose product is -18 and whose sum is -7 are -9 and 2. Thus $3x^2 - 21x - 54 = 3(x - 9)(x + 2)$.
c) Two numbers whose product is 55 and whose sum is -16 are -11 and -5. Thus
$z^2 - 16z + 55 = (z - 11)(z - 5)$.
d) Two numbers whose product is -50 and whose sum is 5 are 10 and -5. Thus
$x^2 + 5x - 50 = (x + 10)(x - 5)$.
e) Since there is an x present in each term it can be factored out: $x^3 - 3x^2 - 10x = x(x^2 - 3x - 10)$. Two numbers whose product is -10 and whose sum is -3 are -5 and 2. Thus $x^3 - 3x^2 - 10x = x(x - 5)(x + 2)$.
f) Since there is a $2x$ present in each term it can be factored out: $2xy^2 - 26yx + 84x = 2x(y^2 - 13y + 42)$.
Two numbers whose product is 42 and whose sum is -13 are -6 and -7. Thus
$2xy^2 - 26yx + 84x = 2x(y - 6)(y - 7)$.

13. a) i) Two numbers whose product is -8 and whose sum is 2 are 4 and -2. The equation in factored form is $y = (x + 4)(x - 2)$.
ii) Determine the zeros by setting each factor equal to zero.

$x + 4 = 0$
$x = -4$;
$x - 2 = 0$
$x = 2$

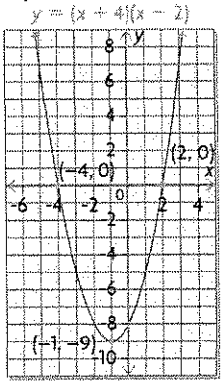
The zeros are -4 and 2.

iii) The x-value halfway between these two zeros is -1. When $x = -1$ then

$y = (x + 4)(x - 2)$
$y = (-1 + 4)(-1 - 2)$
$y = (3)(-3)$
$y = -9$

The vertex is $(-1, -9)$.

iv)



b) i) Two numbers whose product is -24 and whose sum is -2 are 4 and -6. The equation in factored form is $y = (x - 6)(x + 4)$.
ii) Determine the zeros by setting each factor equal to zero.

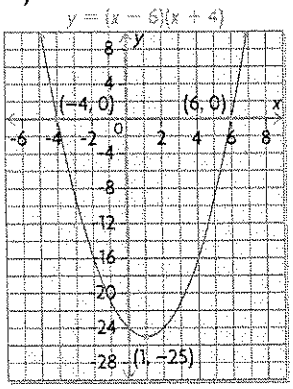
$x - 6 = 0$
$x = 6$;
$x + 4 = 0$
$x = -4$

The zeros are 6 and -4.
iii) The x-value halfway between these two zeros is 1. When $x = 1$ then

$y = (x + 4)(x - 6)$
$y = (1 + 4)(1 - 6)$
$y = (5)(-5)$
$y = -25$

The vertex is $(1, -25)$.

iv)



c) i) Two numbers whose product is 15 and whose sum is -8 are -3 and -5. The equation in factored form is $y = (x - 3)(x - 5)$.
ii) Determine the zeros by setting each factor equal to zero.

$x - 3 = 0$
$x = 3$;
$x - 5 = 0$
$x = 5$

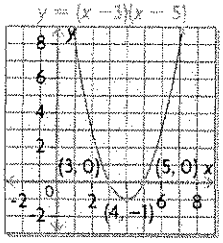
The zeros are 3 and 5.

iii) The x-value halfway between these two zeros is 4. When $x = 4$ then

$y = (x - 3)(x - 5)$
$y = (4 - 3)(4 - 5)$
$y = (1)(-1)$
$y = -1$

The vertex is $(4, -1)$.

iv)



d) i) Since the x^2 term has a coefficient of -1 it must be factored out:

$y = -x^2 - 9x - 14$
$y = -(x^2 + 9x + 14)$

Two numbers whose product is 14 and whose sum is 9 are 7 and 2. The equation in factored form is $y = -(x + 7)(x + 2)$.

ii) Determine the zeros by setting each factor equal to zero.

$x + 7 = 0$
$x = -7$
$x + 2 = 0$
$x = -2$

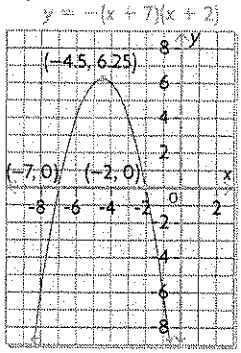
The zeros are -7 and -2.

iii) The x-value halfway between these two zeros is -4.5. When $x = -4.5$ then

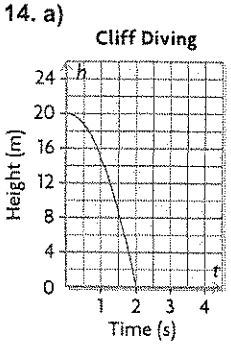
$y = -(x + 7)(x + 2)$
$y = -(-4.5 + 7)(-4.5 + 2)$
$y = -(2.5)(-2.5)$
$y = 6.25$

The vertex is $(-4.5, 6.25)$.

iv)



14. a)



b) The height of the cliff is the height when $t = 0$. The cliff is 20 m high.

c) The time the diver will enter the water is when $h = 0$. The diver will hit the water at 2 s.

15. The maximum height of the ball will occur at the vertex. To determine the vertex first factor the equation.

$h = -5t^2 + 10t + 40$
$h = -5(t^2 - 2t - 8)$
$h = -5(t - 4)(t + 2)$

The zeros are 4 and -2. The t coordinate of the vertex is $t = 0.5(4 - 2) = 1$. When $t = 1$ then

$h = -5t^2 + 10t + 40$
$h = -5(1)^2 + 10(1) + 40$
$h = -5 + 10 + 40$
$h = 45$

The maximum height reached by the ball was 45 m.

16. a) Two monomials whose product is $-5n^2$ and whose sum is $4n$ are $-n$ and $5n$. The factored expression is $(m - n)(m + 5n)$.

b) Two monomials whose product is $35y^2$ and whose sum is $12y$ are $7y$ and $5y$. The factored expression is $(x + 7y)(x + 5y)$.

c) Two monomials whose product is $-12b^2$ and whose sum is b are $-3b$ and $4b$. The factored expression is $(a - 3b)(a + 4b)$.

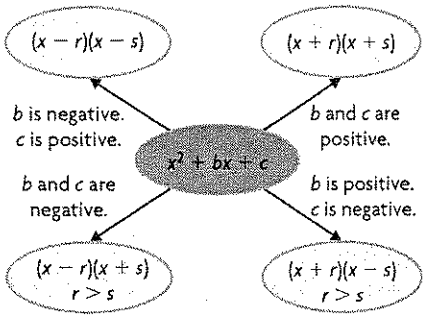
d) Two monomials whose product is $-85d^2$ and whose sum is $-12d$ are $-17d$ and $5d$. The factored expression is $(c - 17d)(c + 5d)$.

e) Two monomials whose product is $12s^2$ and whose sum is $13s$ are s and $12s$. The factored expression is $(r + s)(r + 12s)$.

f) Two monomials whose product is $18p^2$ and whose sum is $-9p$ are $-6p$ and $-3p$. The factored expression is $(q - 6p)(q - 3p)$ or equivalently $(6p - q)(3p - q)$.

17. Agree. Answers may vary, e.g., since c is negative, one factor must be positive and the other must be negative. Suppose these factors are r and $-s$. If $r + (-s) = -b$, then by multiplying each side by -1 we can show that $-r + s = b$. Therefore, the factors for $x^2 - bx - c$ and the factors for $x^2 + bx - c$ have opposite signs. For example, $(x + 2)(x - 4) = x^2 - 2x - 8$, and $(x - 2)(x + 4) = x^2 + 2x - 8$.

18. Answers may vary, e.g.,


(x − r)(x − s)
(x + r)(x + s)
b is negative.
c is positive.
b and c are positive.
x² + bx + c
b and c are negative.
b is positive.
c is negative.
(x − r)(x + s)
r > s
(x + r)(x − s)
r > s

19. a) By thinking of x^4 as $(x^2)^2$, it is possible to factor this expression. Two numbers whose product is –27 and whose sum is 6 are 9 and –3. The factored equation is $(x^2 + 9)(x^2 - 3)$.
b) Two numbers whose product is 9 and whose sum is 10 are 9 and 1. The factored equation is $(a^2 + 9)(a^2 + 1)$.
c) Since m^4 has a coefficient of –4, this must be factored out:
$-4m^4 + 16m^2n^2 + 20n^4 = -4(m^4 - 4m^2n^2 - 5n^4)$
Two monomials whose product is $-5n^4$ and whose sum is $-4n^2$ are $-5n^2$ and n^2. The factored expression is $-4(m^2 - 5n^2)(m^2 + n^2)$.
d) Two numbers whose product is 26 and whose sum is –15 are –13 and –2. The factored expression is $(a - b - 13)(a - b - 2)$.

20. a) $\dfrac{x^2-6x+8}{x-4} = \dfrac{(x-4)(x-2)}{(x-4)}$

$= x-2$

b) $\dfrac{a^2-3a-28}{a+4} = \dfrac{(a+4)(a-7)}{(a+4)}$

$= a-7$

c) $\dfrac{x^2+x-30}{x-5} = \dfrac{(x-5)(x+6)}{(x-5)}$

$= x+6$

d) $\dfrac{2x^2-24x+64}{2x-16} = \dfrac{2(x^2-12x+32)}{2(x-8)}$

$= \dfrac{(x^2-12x+32)}{(x-8)}$

$= \dfrac{(x-8)(x-4)}{(x-8)}$

$= x-4$

Mid-Chapter Review, page 216

1. a) factors of 24: 1, 2, 3, 4, 6, 8, 12, 24;
factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60;
The GCF is 12.
b) The highest degree is 2, The GCF is x^2.
c) factors of 5: 1, 5; factors of 10: 1, 2, 5, 10;
The highest common power is 1, so the GCF is $5y$.
d) factors of –8: –1, –2, –4, –8;
factors of –12: –1, –2, –3, –4, –6, –12;
The greatest common power of both a and b is 1, so the GCF is $-4ab$.
e) The greatest common power of c is 3 and of d is 1.
The GCF is c^3d.
f) factors of 27: 1, 3, 9, 27;
factors of 36: 1, 2, 3, 4, 9, 12, 18, 36;
The greatest common power of both m and n is 2. The GCF is $9m^2n^2$.
2. a) To have $7x$, x must be multiplied by 7. To check, $7(-4y) = -28y$. The missing factor is 7.
b) $(3)(\blacksquare) = 6x - 9y$

$\blacksquare = \dfrac{6x}{3} - \dfrac{9y}{3}$

$\blacksquare = 2x - 3y$

The missing factor is $2x - 3y$.
c) To get $24a^2$, $2a^2$ must be multiplied by 12. To check, $12(b^2) = 12b^2$. The missing factor is 12.
d) $\blacksquare x = x^2 - xy^3$

$\blacksquare = \dfrac{x^2}{x} - \dfrac{xy^3}{x}$

$\blacksquare = x - y^3$

The missing factor is $x - y^3$.
e) To get $-15x^2$, $5x^2$ must be multiplied by –3. To check, $-3(-2y^2) = 6y^2$. The missing factor is –3.
f) $(a^3b^2)\blacksquare = a^4b^3 - a^3b^2$

$\blacksquare = \dfrac{a^4b^3}{a^3b^2} - \dfrac{a^3b^2}{a^3b^2}$

$\blacksquare = ab - 1$

The missing factor is $ab - 1$.
3. a) By counting the algebra tiles the expression is $9x + 6$. The GCF is 3.
b) By counting the algebra tiles the expression is $-4x^2 + 12x$. The GCF is $-4x$.
4. a) $7z + 35 = 7(z + 5)$
b) $-28x^2 + 4x^3 = -4x^2(7 - x)$
c) $5m^2 - 10mn + 5 = 5(m^2 - 2mn + 1)$
d) $x^2y^4 - xy^2 + x^3y = xy(xy^3 - y + x^2)$
5. Answers may vary, e.g., factor the right side of the equation to get $5x(x - 3)$. Determine the zeros from the factors: $x = 0$ and $x = 3$. The x-value of the vertex must be halfway between these two numbers, so the vertex is at $x = 1.5$. Substitute this value into the original equation to determine the y-value of the vertex: $y = 5(1.5)^2 - 15(1.5) = -11.25$. The vertex is (1.5, –11.25).
6. a) $3x(5y - 2) + 5(5y - 2) = (5y - 2)(3x + 5)$
b) $4a(b + 6) - 3(b + 6) = (b + 6)(4a - 3)$
c) $6xt - 2xy - 3t + y = 2x(3t - y) - (3t - y)$

$= (2x - 1)(3t - y)$

d) $4ab + 4ac - b^2 - bc = 4a(b + c) - b(b + c)$

$= (b + c)(4a - b)$

7. a) The quadratic expression is $x^2 + 6x + 8$. Its factors are $(x + 2)$ and $(x + 4)$.
b) The quadratic expression is $x^2 - 2x - 3$. Its factors are $(x - 3)$ and $(x + 1)$.

8. a) $3 \times \blacksquare = 12$
$\blacksquare = 4;$
$\blacklozenge = \blacksquare + 3$
$\blacklozenge = 4 + 3$
$\blacklozenge = 7;$

b) $\blacksquare = 3 + 3$
$\blacksquare = 6;$
$\blacklozenge = 3 \times 3$
$\blacklozenge = 9;$

c) $-\blacklozenge + (-\blacklozenge) = -12$
$-2\blacklozenge = -12$
$\blacklozenge = 6;$
$\blacksquare = \blacklozenge \times \blacklozenge$
$\blacksquare = 6 \times 6$
$\blacksquare = 36;$

d) $-\blacksquare + (-3) = -7$
$-\blacksquare = -4$
$\blacksquare = 4;$
$\blacklozenge = 3 \times \blacksquare$
$\blacklozenge = 3 \times 4$
$\blacklozenge = 12;$

9. a) Two numbers whose product is –33 and whose sum is 8 are –3 and 11. The factored expression is $(x - 3)(x + 11)$.
b) Two numbers whose product is –18 and whose sum is 7 are –2 and 9. The factored expression is $(n - 2)(n + 9)$.
c) Two numbers whose product is –11 and whose sum is –10 are –11 and 1. The factored expression is $(b - 11)(b + 1)$.
d) Two numbers whose product is 45 and whose sum is –14 are –9 and –5. The factored expression is $(x - 9)(x - 5)$.
e) Two numbers whose product is –14 and whose sum is 5 are 7 and –2. The factored expression is $(c + 7)(c - 2)$.
f) Two numbers whose product is 72 and whose sum is –17 are –9 and –8. The factored expression is $(y - 9)(y - 8)$.

10. a) $3a^2 - 3a - 36 = 3(a^2 - a - 12)$
$= 3(a - 4)(a + 3)$

b) $x^3 - 6x^2 - 16x = x(x^2 - 6x - 16)$
$= x(x - 8)(x + 2)$

c) $2x^2 + 14x - 120 = 2(x^2 + 7x - 60)$
$= 2(x + 12)(x - 5)$

d) $4b^2 - 36b + 72 = 4(b^2 - 9b + 18)$
$= 4(b - 6)(b - 3)$

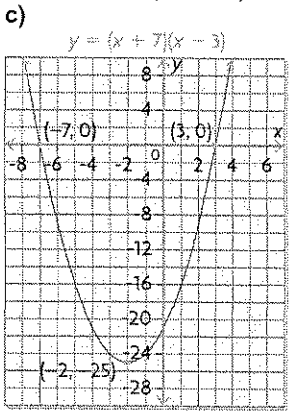
e) $-d^3 + d^2 + 30d = -d(d^2 - d - 30)$
$= -d(d - 6)(d + 5)$

f) $xy^3 + 2xy^2 + xy = xy(y^2 + 2y + 1)$
$= xy(y + 1)(y + 1)$

11. a) The height of the cliff is h when $t = 0$.
$h = -5t^2 + 10t + 75$
$h = -5(0) + 10(0) + 75$
$h = 75$
The cliff is 75 m high.
b) Factor the equation:
$h = -5t^2 + 10t + 75$
$h = -5(t^2 - 2t - 15)$
$h = -5(t - 5)(t + 3)$
The two zeros are 5 and –3. The maximum height will be reached in the middle of these two points.
$x = 0.5(-3 + 5)$
$x = 0.5(2)$
$x = 1$
The maximum height will be reached at 1 s.
c) From part **b),** the maximum height is the height at $t = 1$,
$h = -5t^2 + 10t + 75$
$h = -5(1)^2 + 10(1) + 75$
$h = -5 + 10 + 75$
$h = 80$
The maximum height is 80 m.

12. Answers may vary, e.g., there are usually fewer factors for c to consider than there are addends for b, so it makes sense to consider the value of c first. This allows you to use a guess-and-check strategy more efficiently. Using $x^2 + 6x + 5$ as an example, there are many pairs of numbers that add up to 6 (3, 3; 2, 4; 1, 5; 0, 6; –1, 7; etc.) but only one pair that multiply together to equal 5 (1, 5).

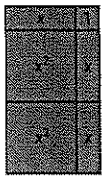
13. a) Two numbers whose product is –21 and whose sum is 4 are 7 and –3. The factored expression is $y = (x + 7)(x - 3)$
b) By setting each factor equal to 0, the zeros are $x = -7$ and $x = 3$. The x-value of the vertex is midway between these points,
$x = 0.5(-7 + 3)$
$x = -2.$
When $x = -2$,
$y = x^2 + 4x - 21$
$y = (-2)^2 + 4(-2) - 21$
$y = 4 - 8 - 21$
$y = -25$
The vertex is (–2, –25).
c)



Lesson 4.4: Factoring Quadratics: $ax^2 + bx + c$, page 222

1. a) The trinomial represented by the tiles is $2x^2 + 3x + 1$.

b)



c) The factors of this trinomial are $(2x + 1)$ and $(x + 1)$.

2. a) The trinomial is $2x^2 + 5x + 2$ and its factors are $(2x + 1)$ and $(x + 2)$.

b) The trinomial is $6x^2 - 5x + 1$ and its factors are $(2x - 1)$ and $(3x - 1)$.

c) The trinomial is $8x^2 + 26x + 15$ and its factors are $(2x + 5)$ and $(4x + 3)$.

d) The trinomial is $15x^2 + 13x - 6$ and its factors are $(3x - 1)$ and $(5x + 6)$.

3. a) The factor is of the form $(qc + s)$.

$4 \times s = -4$

$s = -1$;

$1 \times q = 2$

$q = 2$

The missing factor is $2c - 1$.

b) The factor is of the form $(qz + s)$.

$-9 \times s = -3$

$s = 3$;

$1 \times q = 4$

$q = 4$

The missing factor is $4z + 3$.

c) The factor is of the form $(qy + s)$.

$1 \times s = -1$

$s = -1$;

$3 \times q = 6$

$q = 2$

The missing factor is $2y - 1$.

d) The factor is of the form $(qp + s)$.

$3 \times s = -3$

$s = -1$;

$2 \times q = 6$

$q = 3$

The missing factor is $3p - 1$.

4. a) $3 \times \blacksquare = 3$

$\blacksquare = 1$;

$\blacklozenge x = x \times 1 + 3 \times 5x$

$\blacklozenge = 16$

b) $-\blacklozenge = 3 \times (-2)$

$\blacklozenge = 6$;

$-\blacksquare x = 2x \times (-2) + 3 \times x$

$\blacksquare = 1$

c) $-7x = 3x(-\blacklozenge) + 4x(-\blacklozenge)$

$= -7x\blacklozenge$

$\blacklozenge = 1$;

$\blacksquare = -1 \times (-1)$

$\blacksquare = 1$

d) $-29x = 2x(-\blacksquare) - 3(7x)$

$= -2x(\blacksquare) - 21x$

$-8x = -2x(\blacksquare)$

$\blacksquare = 4$;

$\blacklozenge = 4 \times (-3)$

$\blacklozenge = 12$

5. a) Two numbers whose product is $-6 \times 2 = -12$ and whose sum is 1 are 4 and -3.

$$2x^2 + x - 6 = 2x^2 - 3x + 4x - 6$$
$$= x(2x - 3) + 2(2x - 3)$$
$$= (2x - 3)(x + 2)$$

b) Two numbers whose product is $-4 \times 3 = -12$ and whose sum is -11 are 1 and -12.

$$3n^2 - 11n - 4 = 3n^2 - 12n + 1n - 4$$
$$= 3n(n - 4) + 1(n - 4)$$
$$= (3n + 1)(n - 4)$$

c) Two numbers whose product is $-1 \times 10 = -10$ and whose sum is 3 are 5 and -2.

$$10a^2 + 3a - 1 = 10a^2 + 5a - 2a - 1$$
$$= 5a(2a + 1) - 1(2a + 1)$$
$$= (5a - 1)(2a + 1)$$

d) Two numbers whose product is $4 \times 15 = 60$ and whose sum is -16 are -6 and -10.

$$4x^2 - 16x + 15 = 4x^2 - 6x - 10x + 15$$
$$= 2x(2x - 3) - 5(2x - 3)$$
$$= (2x - 5)(2x - 3)$$

e) Two numbers whose product is $-12 \times 2 = -24$ and whose sum is 5 are 8 and -3.

$$2c^2 + 5c - 12 = 2c^2 - 3c + 8c - 12$$
$$= c(2c - 3) + 4(2c - 3)$$
$$= (2c - 3)(c + 4)$$

f) Two numbers whose product is $6 \times 1 = 6$ and whose sum is 5 are 3 and 2.

$$6x^2 + 5x + 1 = 6x^2 + 2x + 3x + 1$$
$$= 2x(3x + 1) + 1(3x + 1)$$
$$= (3x + 1)(2x + 1)$$

6. a) Two numbers whose product is $6 \times 6 = 36$ and whose sum is -13 are -9 and -4.

$$6x^2 - 13x + 6 = 6x^2 - 9x - 4x + 6$$
$$= 3x(2x - 3) - 2(2x - 3)$$
$$= (3x - 2)(2x - 3)$$

b) Two numbers whose product is $-3 \times 10 = -30$ and whose sum is 1 are 6 and -5.

$$10m^2 + m - 3 = 10m^2 - 5m + 6m - 3$$
$$= 5m(2m - 1) + 3(2m - 1)$$
$$= (2m - 1)(5m + 3)$$

c) Two numbers whose product is $2 \times 12 = 24$ and whose sum is -11 are -3 and -8.

$$2a^2 - 11a + 12 = 2a^2 - 3a - 8a + 12$$
$$= a(2a - 3) - 4(2a - 3)$$
$$= (2a - 3)(a - 4)$$

d) Two numbers whose product is $4 \times 25 = 100$ and whose sum is -20 are -10 and -10.

$$4x^2 - 20x + 25 = 4x^2 - 10x - 10x + 25$$
$$= 2x(2x - 5) - 5(2x - 5)$$
$$= (2x - 5)(2x - 5)$$

e) Two numbers whose product is $5 \times 8 = 40$ and whose sum is -14 are -10 and -4.

$$\begin{aligned} 5d^2 - 14d + 8 &= 5d^2 - 10d - 4d + 8 \\ &= 5d(d - 2) - 4(d - 2) \\ &= (5d - 4)(d - 2) \end{aligned}$$

f) Two numbers whose product is $3 \times (-10) = -30$ and whose sum is 13 are -2 and 15.

$$\begin{aligned} 6n^2 + 26n - 20 &= 2(3n^2 + 13n - 10) \\ &= 2(3n^2 - 2n + 15n - 10) \\ &= 2[n(3n - 2) + 5(3n - 2)] \\ &= 2(3n - 2)(n + 5) \end{aligned}$$

7. a) Two numbers whose product is $15 \times (-4) = -60$ and whose sum is 4 are -6 and 10.

$$\begin{aligned} 15x^2 + 4x - 4 &= 15x^2 - 6x + 10x - 4 \\ &= 3x(5x - 2) + 2(5x - 2) \\ &= (5x - 2)(3x + 2) \end{aligned}$$

b) Two numbers whose product is $18 \times (-10) = -180$ and whose sum is -3 are 12 and -15.

$$\begin{aligned} 18m^2 - 3m - 10 &= 18m^2 - 15m + 12m - 10 \\ &= 3m(6m - 5) + 2(6m - 5) \\ &= (6m - 5)(3m + 2) \end{aligned}$$

c) Two numbers whose product is $8 \times 18 = 144$ and whose sum is -25 are -9 and -16.

$$\begin{aligned} 16a^2 - 50a + 36 &= 2(8a^2 - 25a + 18) \\ &= 2(8a^2 - 16a - 9a + 18) \\ &= 2[8a(a - 2) - 9(a - 2)] \\ &= 2(a - 2)(8a - 9) \end{aligned}$$

d) Two numbers whose product is $35 \times (-18) = -630$ and whose sum is -27 are -42 and 15.

$$\begin{aligned} 35x^2 - 27x - 18 &= 35x^2 - 42x + 15x - 18 \\ &= 7x(5x - 6) + 3(5x - 6) \\ &= (7x + 3)(5x - 6) \end{aligned}$$

e) $63n^2 + 126n + 48 = 3(21n^2 + 42n + 16)$
Since there are no two numbers whose product is $21 \times 16 = 336$ and whose sum is 42, this expression cannot be factored further.

f) Two numbers whose product is $24 \times 35 = 840$ and whose sum is -62 are -42 and -20.

$$\begin{aligned} 24d^2 - 62d + 35 &= 24d^2 - 42d - 20d + 35 \\ &= 6d(4d - 7) - 5(4d - 7) \\ &= (4d - 7)(6d - 5) \end{aligned}$$

8. Answers may vary, e.g.,
$3x^2 - x - 4 = (3x - 4)(x + 1)$;
$3x^2 - 13x + 12 = (3x - 4)(x - 3)$;
$6x^2 + 13x - 28 = (3x - 4)(2x + 7)$

9. Since the area of a rectangle is length times width, factor the trinomial and let one factor be length and the other be width.

a) Two numbers whose product is $6 \times (-3) = -18$ and whose sum is 17 are -1 and 18.

$$\begin{aligned} A &= 6x^2 + 17x - 3 \\ A &= 6x^2 + 18x - 1x - 3 \\ A &= 6x(x + 3) - 1(x + 3) \\ A &= (6x - 1)(x + 3) \end{aligned}$$

The length is $(6x - 1)$ and the width is $(x + 3)$, or vice versa.

b) Two numbers whose product is $8 \times 15 = 120$ and whose sum is -26 are -6 and -20.

$$\begin{aligned} A &= 8x^2 - 26x + 15 \\ A &= 8x^2 - 6x - 20x + 15 \\ A &= 2x(4x - 3) - 5(4x - 3) \\ A &= (4x - 3)(2x - 5) \end{aligned}$$

The length is $(4x - 3)$ and the width is $(2x - 5)$, or vice versa.

10. Answers may vary, e.g.,

a) Let the factors be $(px + r)$ and $(qx + s)$. $r \times s = 2$, so r and s must be either 1 and 2, or -1 and -2. This gives $p + 2q = 5$ or $-p - 2q = 5$. Possible values of p and q that satisfy one of these equations include 1 and 2, 3 and 1, or 1 and -3. Since $k = p \times q$, possible k-values include $k = 2$, 3, or -3.

b) To use the decomposition strategy, decompose k into two numbers whose product is $9 \times (-5) = -45$. Possible pairs include 9 and -5, 15 and -3, 45 and -1, -15 and 3, or -9 and 5. By adding each pair, we see that possible k-values include $k = 4$, 12, 44, -12, -4, or -44.

c) Let the factors be $(px + r)$ and $(qx + s)$. $p \times q = 12$, so p and q must be 1 and 12, 2 and 6, or 3 and 4. This gives $s + 12r = -20$, $2s + 6r = -20$, or $3s + 4r = -20$. Possible values of s and r that satisfy one of these equations include -1 and -3, -7 and -1, -4 and -3, or 4 and -2. Since $k = s \times r$, possible k-values are $k = 3$, 7, 8, or -8.

11. a) Two numbers whose product is $3 \times (-6) = -18$ and whose sum is 17 are 18 and -1.

$$\begin{aligned} 6x^2 + 34x - 12 &= 2(3x^2 + 17x - 6) \\ &= 2(3x^2 + 18x - 1x - 6) \\ &= 2(3x - 1)(x + 6) \end{aligned}$$

b) Two numbers whose product is $6 \times (-10) = -60$ and whose sum is 11 are 15 and -4.

$$\begin{aligned} 18v^2 + 33v - 30 &= 3(6v^2 + 11v - 10) \\ &= 3(6v^2 + 15v - 4v - 10) \\ &= 3[3v(2v + 5) - 2(2v + 5)] \\ &= 3(2v + 5)(3v - 2) \end{aligned}$$

c) Two numbers whose product is $12 \times 25 = 300$ and whose sum is -40 are -30 and -10.

$$\begin{aligned} 48c^2 - 160c + 100 &= 4(12c^2 - 40c + 25) \\ &= 4(12c^2 - 30c - 10c + 25) \\ &= 4[6c(2c - 5) - 5(2c - 5)] \\ &= 4(2c - 5)(6c - 5) \end{aligned}$$

d) Two numbers whose product is $5 \times 6 = 30$ and whose sum is -17 are -15 and -2.

$$\begin{aligned} 5b^3 - 17b^2 + 6b &= b(5b^2 - 17b + 6) \\ &= b(5b^2 - 15b - 2b + 6) \\ &= b[5b(b - 3) - 2(b - 3)] \\ &= b(5b - 2)(b - 3) \end{aligned}$$

e) Two numbers whose product is $9 \times (-2) = -18$ and whose sum is -17 are 1 and -18.

$$\begin{aligned} 27xy^2 - 51xy - 6x &= 3x(9y^2 - 17y - 2) \\ &= 3x(9y^2 + y - 18y - 2) \\ &= 3x[y(9y + 1) - 2(9y + 1)] \\ &= 3x(9y + 1)(y - 2) \end{aligned}$$

f) Two numbers whose product is $7 \times (-30) = -210$ and whose sum is 29 are 35 and –6.

$$\begin{aligned} -7a^2 - 29a + 30 &= -(7a^2 + 29a - 30) \\ &= -(7a^2 + 35b - 6a - 30) \\ &= -[7a(a + 5) - 6(a + 5)] \\ &= -(a + 5)(7a - 6) \end{aligned}$$

12. a) Two numbers whose product is –52 and whose sum is 9 are 13 and –4.

$k^2 + 9k - 52 = (k + 13)(k - 4)$

No, $(k + 5)$ is not a factor of this polynomial.

b) First divide out the common factor of $4k$. Two numbers whose product is 15 and whose sum is 8 are 3 and 5.

$$\begin{aligned} 4k^3 + 32k^2 + 60k &= 4k(k^2 + 8k + 15) \\ &= 4k(k + 3)(k + 5) \end{aligned}$$

Yes, $(k + 5)$ is a factor of this polynomial.

c) Two numbers whose product is $6 \times 7 = 42$ and whose sum is 23 are 21 and 2.

$$\begin{aligned} 6k^2 + 23k + 7 &= 6k^2 + 21k + 2k + 7 \\ &= 3k(2k + 7) + 1(2k + 7) \\ &= (2k + 7)(3k + 1) \end{aligned}$$

No, $(k + 5)$ is not a factor of this polynomial.

d) First divide out the common factor of –1. Two numbers whose product is $15 \times (-10) = 150$ and whose sum is –19 are 6 and –25.

$$\begin{aligned} -15k^2 + 19k + 10 &= -(15k^2 - 19k - 10) \\ &= -(15k^2 + 6k - 25k - 10) \\ &= -[3k(5k + 2) - 5[5k + 2)] \\ &= -(3k - 5)(5k + 2) \end{aligned}$$

No, $(k + 5)$ is not a factor of this polynomial.

e) Two numbers whose product is $7 \times (-30) = -210$ and whose sum is 29 are –6 and 35.

$$\begin{aligned} 7k^2 + 29k - 30 &= 7k^2 - 6k + 35k - 30 \\ &= k(7k - 6) + 5(7k - 6) \\ &= (7k - 6)(k + 5) \end{aligned}$$

Yes, $(k + 5)$ is a factor of this polynomial.

f) First divide out the common factor of 5. Two numbers whose product is $2 \times 15 = 30$ and whose sum is 13 are 3 and 10.

$$\begin{aligned} 10k^2 + 65k + 75 &= 5(2k^2 + 13k + 15) \\ &= 5(2k^2 + 3k + 10k + 15) \\ &= 5[k(2k + 3) + 5(2k + 3)] \\ &= 5(k + 5)(2k + 3) \end{aligned}$$

Yes, $(k + 5)$ is a factor of this polynomial.

Polynomials b), e), and f) have $(k + 5)$ as a factor.

13. a) i) Two numbers whose product is $2 \times 4 = 8$ and whose sum is –9 are –1 and –8.

$$\begin{aligned} y &= 2x^2 - 9x + 4 \\ &= 2x^2 - 1x - 8x + 4 \\ &= x(2x - 1) - 4(2x - 1) \\ &= (2x - 1)(x - 4) \end{aligned}$$

ii) Set each factor equal to 0:

$$\begin{aligned} 2x - 1 &= 0 & \qquad x - 4 &= 0 \\ 2x &= 1 & \qquad x &= 4 \\ x &= 0.5 \end{aligned}$$

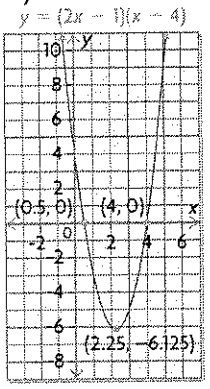
The zeros are 0.5 and 4.

iii) The x-value of the vertex is midway between the zeros, $x = 2.25$. When $x = 2.25$,

$$\begin{aligned} y &= 2x^2 - 9x + 4 \\ &= 2(2.25)^2 - 9(2.25) + 4 \\ &= -6.125 \end{aligned}$$

The vertex is (2.25, –6.125).

iv)



b) i) Two numbers whose product is $-2 \times 15 = -30$ and whose sum is 7 are 10 and –3.

$$\begin{aligned} y &= -2x^2 + 7x + 15 \\ &= -2x^2 + 10x - 3x + 15 \\ &= -2x(x - 5) - 3(x - 5) \\ &= -(2x + 3)(x - 5) \text{ or } (2x + 3)(-x + 5) \end{aligned}$$

ii) Set each factor equal to 0:

$$\begin{aligned} 2x + 3 &= 0 \\ 2x &= -3 \\ x &= -1.5 \\ x - 5 &= 0 \\ x &= 5 \end{aligned}$$

The zeros are –1.5 and 5.

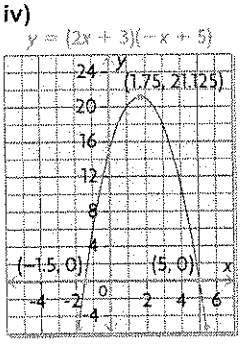
iii) The x-value of the vertex is midway between the zeros, $x = 0.5(-1.5 + 5) = 1.75$.

When $x = 1.75$,

$$\begin{aligned} y &= -2(1.75)^2 + 7(1.75) + 15 \\ y &= -6.125 + 12.25 + 15 \\ y &= 21.125 \end{aligned}$$

The vertex is (1.75, 21.125).

iv)



14. a) The break-even points are the zeros of the profit equation.

$P = -4x^2 + 20x - 9$
$P = -(4x^2 - 20x + 9)$
$P = -(4x^2 + 18x + 2x + 9)$
$P = -[2x(2x + 9) + 1(2x + 9)]$
$P = (1 - 2x)(2x - 9)$

The break-even points are 0.5 hundred thousand games and 4.5 hundred thousand games.

b) The maximum profit is the y-value of the vertex of the profit equation. The x-value of the vertex is midway between the zeros, $x = 0.5(0.5 + 4.5) = 2.5$. When $x = 2.5$,

$P = -4x^2 + 20x - 9$
$P = -4(2.5)^2 + 20(2.5), - 9$
$P = -25 + 50 - 9$
$P = 16$

The maximum profit is \$16 million.

c) The number of games sold at maximum profit is the x-value of the vertex, 2.5 hundred thousand games or 250 000 games.

15. a) Two numbers whose product is $8 \times 5 = 40$ and whose sum is -13 are -5 and -8.

$8x^2 - 13xy + 5y^2 = 8x^2 - 5xy - 8xy + 5y^2$
$= x(8x - 5y) - y(8x - 5y)$
$= (8x - 5y)(x - y)$

b) Two numbers whose product is $5 \times 6 = 30$ and whose sum is -17 are -15 and -2.

$5a^2 - 17ab + 6b^2 = 5a^2 - 15ab - 2ab + 6b^2$
$= 5a(a - 3b) - 2b(a - 3b)$
$= (5a - 2b)(a - 3b)$

c) First divide out the common factor of -1. Two numbers whose product is $12 \times (-35) = -420$ and whose sum is 1 are 21 and -20.

$-12s^2 - sr + 35r^2 = -(12s^2 + 21sr - 20sr - 35r^2)$
$= -3s(4s + 7r) - 5r(4s + 7r)$
$= -(4s + 7r)(3s - 5r)$

d) Two numbers whose product is $16 \times 39 = 624$ and whose sum is 64 are 52 and 12.

$16c^4 + 64c^2 + 39 = 16c^4 + 52c^2 + 12c^2 + 39$
$= 4c^2(4c^2 + 13) + 3(4c^2 + 13)$
$= (4c^2 + 3)(4c^2 + 13)$

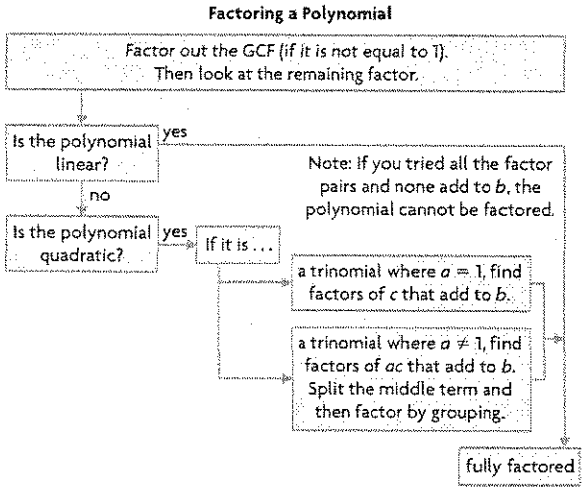
e) Two numbers whose product is $14 \times 27 = 378$ and whose sum is -39 are -21 and -18.

$14v^6 - 39v^3 + 27 = 14v^6 - 21v^3 - 18v^3 + 27$
$= 7v^3(2v^3 - 3) - 9(2v^3 - 3)$
$= (7v^3 - 9)(2v^3 - 3)$

f) First divide out the common factor cd. Two numbers whose product is $1 \times (-8) = -8$ and whose sum is 2 are 4 and -2.

$c^3d^3 + 2c^2d^2 - 8cd = cd(c^2d^2 + 2cd - 8)$
$= cd(c^2d^2 + 4cd - 2cd - 8)$
$= cd[cd(cd + 4) - 2(cd + 4)]$
$= cd(cd - 2)(cd + 4)$

16. Answers may vary, e.g.,


Factoring a Polynomial
Factor out the GCF (if it is not equal to 1). Then look at the remaining factor.
Is the polynomial linear?
yes
no
Is the polynomial quadratic?
yes
If it is . . .
Note: If you tried all the factor pairs and none add to b, the polynomial cannot be factored.
a trinomial where a = 1, find factors of c that add to b.
a trinomial where a ≠ 1, find factors of ac that add to b. Split the middle term and then factor by grouping.
fully factored

17. a) $6(a + b)^2 + 11(a + b) + 3$
$= 6(a + b)^2 + 9(a + b) + 2(a + b) + 3$
$= 3(a + b)[2(a + b) + 3] + 1[2(a + b) + 3]$
$= [3(a + b) + 1][2(a + b) + 3]$
$= (3a + 3b + 1)(2a + 2b + 3)$

b) $5(x - y)^2 - 7(x - y) - 6$
$= 5(x - y)^2 - 10(x - y) + 3(x - y) - 6$
$= 5(x - y)[(x - y) - 2] + 3[(x - y) - 2]$
$= [5(x - y) + 3)][(x - y) - 2]$
$= (5x - 5y + 3)(x - y - 2)$

c) $8(x + 1)^2 - 14(x + 1) + 3$
$= 8(x + 1)^2 - 12(x + 1) - 2(x + 1) + 3$
$= 4(x + 1)[2(x + 1) - 3] - 1[2(x + 1) - 3]$
$= [4(x + 1) - 1][2(x + 1) - 3]$
$= (4x + 3)(2x - 1)$

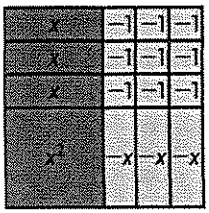
d) Let $x = (a - 2)^2 = a^2 - 4a + 4$
$12(a - 2)^4 + 52(a - 2)^2 - 40$
$= 12x^2 + 52x - 40$
$= 4(3x^2 + 13x - 10)$
$= 4(3x - 2)(x + 5)$
$= 4[3(a^2 - 4a + 4) - 2][(a^2 - 4a + 4) + 5]$
$= 4(3a^2 - 12a + 10)(a^2 - 4a + 4 + 5)$
$= 4(3a^2 - 12a + 12 - 2)(a^2 - 4a + 9)$
$= 4(3a^2 - 12a + 10)(a^2 - 4a + 9)$

18. Yes. Answers may vary, e.g., if $b^2 - 4ac$ is a perfect square, then the expression will be factorable. For example, $6x^2 + 11x + 3$ implies that $b^2 - 4ac = 11^2 - 4(6)(3) = 49$. This is a perfect square, and the expression is factorable as $(3x + 1)(2x + 3)$.

Lesson 4.5: Factoring Quadratics: Special Cases, page 229

1. a) There are nine –1 tiles, three x tiles, three $-x$ tiles, and one x^2 tile. The three x tiles and the three $-x$ tiles have a sum of 0, so the equation represented is $x^2 - 9$.

b)



c) The factors of this trinomial are $(x + 3)$ and $(x - 3)$.

2. a) The equation of the polynomial is $9x^2 - 4$, and its factors are $(3x - 2)$ and $(3x + 2)$.

b) The equation of the polynomial is $36x^2 + 60x + 25$ and its factors are $(6x + 5)(6x + 5)$ or $(6x + 5)^2$.

3. a) This expression is a difference of squares, with one factor $(x + 10)$. This a square trinomial with a positive middle term, with one factor of $(n + 5)$. The missing term is $n + 5$.

c) This expression is a difference of squares with one factor of $(9a - 4)$. Thus the missing factor is $9a + 4$.

d) When the factors are expanded I get $4x^2 - 1$. To get the original equation this expression must be multiplied by 5, The missing factor is 5.

e) The square root of the first term is $5m$ and the square root of the constant term is 7. Since the middle term is negative then the sign between them must be negative. The missing factor is $5m - 7$.

f) Begin by factoring out a 2 to give $9x^2 - 24x + 16$. Now the square root of the first term is $3x$ and the square root of the constant term is 4. Since the middle term is negative the sign between the terms must be negative. The missing factor is $3x - 4$.

4. a) $■^2 = 25$, so $■ = 5$ or $■ = -5$.

If $■ = 5$, then:

$◆ = 2 \times 5 + 2 \times 5$

$= 20$

If $■ = -5$, then:

$◆ = 2 \times (-5) + 2 \times (-5)$

$= -20$

b) $■^2 = 25$, so $■ = 5$ or $■ = -5$.

$-9 = -3 \times 3$

$◆ = 9$

c) $■^2 = 81$, so $■ = 9$ or $■ = -9$

If $■ = 9$, then:

$◆ = 4 \times 9 + 4 \times 9$

$= 72$

If $■ = -9$, then:

$◆ = 4 \times (-9) + 4 \times (-9)$

$= -72$

d) $◆ = 3 \times 3$

$= 9$

$■^2 = 64$, so $■ = 8$ or $■ = -8$

5. Each of the following expressions is a difference of squares. The factor will be in the form $(\sqrt{a}x + \sqrt{c})(\sqrt{a}x - \sqrt{c})$.

a) $x^2 - 25 = (x - 5)(x + 5)$

b) $y^2 - 81 = (y + 9)(y - 9)$

c) $a^2 - 36 = (a - 6)(a + 6)$

d) $4c^2 - 49 = (2c - 7)(2c + 7)$

e) $9x^2 - 4 = (3x - 2)(3x + 2)$

f) $25d^2 - 144 = (5d - 12)(5d + 12)$

6. Since the a and c values of each of these trinomials are perfect squares, and $b = \pm 2\sqrt{ac}$, then the trinomial is a perfect square. The factor will be in the form $(\sqrt{a}x + \sqrt{c})^2$ or $(\sqrt{a}x - \sqrt{c})^2$, depending on the sign of the middle term.

a) $x^2 + 10x + 25 = (x + 5)^2$

b) $b^2 + 8b + 16 = (b + 4)^2$

c) $m^2 - 4m + 4 = (m - 2)^2$

d) $4c^2 - 44c + 121 = (2c - 11)^2$

e) $16p^2 + 72p + 81 = (4p + 9)^2$

f) $25z^2 - 30z + 9 = (5z - 3)^2$

7. a) Since 49 and 16 are perfect squares and $56 = 2(7)(4)$, this expression is a perfect-square trinomial. The square root of the first term is $7a$ and the square root of the last term is 4. The middle term is positive The two terms will be separated by a plus sign. The factored expression is $(7a + 4)^2$.

b) Since 4 and 25 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is $2x$ and the square root of the second term is 5. Thus the expression in factored form is $(2x - 5)(2x + 5)$.

c) If –2 is factored out the expression is $-2(25x^2 + 20x + 4)$. Since 25 and 4 are perfect squares and $20 = 2(5)(2)$, this expression is a perfect-square trinomial. The square root of the first term is $5x$ and the square root of the last term is 2. The middle term is positive The two terms will be separated by a plus sign. The factored expression is $-2(5x + 2)^2$.

d) If 4 is factored out the expression is $4(a^2 - 64)$. Since 1 and 64 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The root of the first term is a and the root of the second term is 8. Thus the expression in factored form is $4(a - 8)(a + 8)$.

e) Since 225 and 16 are perfect squares and they are separated by a negative sign, the polynomial is a difference of squares. The root of the first term is 15 and the root of the second term is $4x$. Thus the expression in factored form is $(15 - 4x)(15 + 4x)$.

f) Since 1 and 1 are perfect squares and $2 = 2(1)(1)$, this expression is a perfect-square trinomial. The root of the first term is $1(x + 1)$ and the root of the last term is 1. The middle term is positive The two terms will be separated by a plus sign. The factored expression is $(x + 1 + 1)^2 = (x + 2)^2$.

8. a) $64^2 - 60^2 = (64 - 60)(64 + 60)$
$= 4 \times 124$
$= 496$

b) $18^2 - 12^2 = (18 - 12)(18 + 12)$
$= 6 \times 30$
$= 180$

9. a) Answers may vary, e.g., this cannot be factored as a perfect square because the coefficient of x^2 is not a perfect square.

b) Answers may vary, e.g., this cannot be factored as a difference of squares because the coefficient of x^2 is not a perfect square.

10. a) Since 1 and 36 are perfect squares and $12 = 2(1)(6)$, this expression is a perfect-square trinomial. The square root of the first term is x^2 and the square root of the last term is 6. The middle term is negative, The two terms will be separated by a subtraction sign. The factored expression is $(x^2 - 6)^2$.

b) Since 1 and 16 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is a^2 and the square root of the second term is 4. Thus the expression in factored form is $(a^2 + 4)(a^2 - 4)$. Now this expression can be factored further since $(a^2 - 4)$ is also a difference of squares. The square root of the first term is a and the square root of the second term is 2. The expression in factored form is $(a - 2)(a + 2)(a^2 + 4)$.

c) Since 49 and 100 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is $7x$ and the square root of the second term is 10. Thus the expression in factored form is $(7x - 10)(7x + 10)$.

d) When 3 is factored out, the expression is $3(4x^2 - 20x + 25)$. Since 4 and 25 are perfect squares and $20 = (2)(2)(5)$, this expression is a perfect-square trinomial. The square root of the first term is $2x$ and the square root of the last term is 5. The middle term is negative, The two terms will be separated by a subtraction sign. The factored expression is $3(2x - 5)^2$.

e) Since 1 and 144 are perfect squares and $24 = 2(1)(12)$, this expression can be factored into one bracket. The square root of the first term is x^2 and the square root of the last term is 12. The sign of the middle term is negative, The two terms will be separated by a subtraction sign. The factored expression is $(x^2 - 12)^2$.

f) Since 289 and 81 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is $17x^3$ and the square root of the second term is 9. Thus the expression in factored form is $(17x^3 - 9)(17x^3 + 9)$.

11. a) Since 1 and 64 are perfect squares and $16 = 2(1)(8)$, this expression is a difference of squares. The square root of the first term is x and the square root of the last term is $8y$. The middle term is negative, The two terms will be separated by a subtraction sign. The factored expression is $(x - 8y)^2$.

b) Since 36 and 25 are perfect squares and separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is $6x$ and the square root of the second term is $5y$. The factored expression is $(6x - 5y)(6x + 5y)$.

c) Since 16 and 81 are perfect squares and $72 = 2(4)(9)$, this expression is a perfect-square trinomial. The square root of the first term is $4x$ and the square root of the last term is $9y$. The middle term is negative, The two terms will be separated by a subtraction sign. The factored expression is $(4x - 9y)^2$.

d) Since 1 and 9 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is 1 and the square root of the second term is $3ab^2$. Thus the expression in factored form is $(1 - 3ab^2)(1 + 3ab^2)$.

e) When -2 is factored out, the expression is $-2(9x^2 - 12xy + 4y^2)$. Since 9 and 4 are perfect squares and $12 = (2)(3)(2)$, this expression is a perfect-square trinomial. The square root of the first term is $3x$ and the square root of the second term is $2y$. The middle term is negative, The two terms will be separated by a subtraction sign. Thus the expression in factored form is $-2(3x - 2y)^2$.

f) When $2x$ is factored out, the expression is $2x(25x^2 - 4y^2)$. Since 25 and 4 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is $5x$ and the square root of the second term is $2y$. Thus the expression in factored form is $2x(5x - 2y)(5x + 2y)$.

12. a) Answers may vary, e.g., first I noticed $x^2 - 8x + 16$ could be factored as a perfect square. Second, I noticed the result, $(x - 4)^2 - c^2$, could be factored as a difference of squares, resulting in $(x - 4 - c)(x - 4 + c)$.

b) Answers may vary, e.g., first I noticed I could factor out a -1 term in $-a^2 - 6ab - 9b^2$, so the trinomial could be factored as a perfect square. Second, I noticed the result, $4c^2 - (a + 3b)^2$, could be factored as a difference of squares. The expression when completely factored is $(2c - a - 3b)(2c + a + 3b)$.

13. a) i) First factor out -1, the equation is now $y = -(x^2 - 16x + 64)$. Since 1 and 64 are perfect squares and $16 = 2(1)(8)$, this equation is a perfect square. The square root of the first term is x and the square root of the last term is 8. The middle term is negative, The two terms will be separated by a subtraction sign. The factored equation is $y = -(x - 8)^2$.

ii) When $y = 0$ then $x - 8 = 0$
$x = 8$

The zero is 8.

iii) When $x = 8$ then

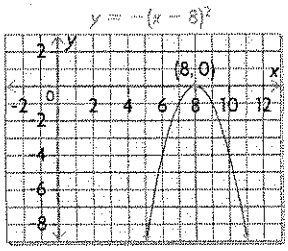
$y = -(x - 8)^2$

$y = -(8 - 8)^2$

$y = 0$

The vertex is (8, 0).

iv)



b) i) Since 4 and 1 are perfect squares and they are separated by a subtraction sign, the polynomial is a difference of squares. The square root of the first term is $2x$ and the square root of the second term is 1. Thus the equation in factored form is $y = (2x - 1)(2x + 1)$.

ii) Calculate the zeros.

$2x - 1 = 0$ $\qquad$ $2x + 1 = 0$

$2x = 1$ $\qquad$ $2x = -1$

$x = 0.5$ $\qquad$ $x = -0.5$

The zeros are $x = 0.5$ and $x = -0.5$.

iii) The x-value of the vertex will be between these zeros, $x = 0.5(0.5 - 0.5) = 0$.

When $x = 0$ then,

$y = (2x - 1)(2x + 1)$

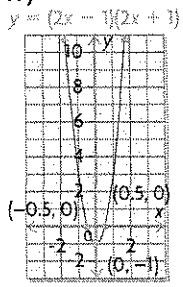
$y = [2(0) - 1][2(0) + 1]$

$y = (-1)(1)$

$y = -1$

The vertex is (0, −1).

iv)



14. a) The area of the shaded region will be the area of the large box minus the area of the small box.

$A = (4x + 5)^2 - (x - 2)^2$

$= (4x + 5)(4x + 5) - (x - 2)(x - 2)$

$= 16x^2 + 40x + 25 - (x^2 - 4x + 4)$

$= 16x^2 + 40x + 25 - x^2 + 4x - 4$

$= 15x^2 + 44x + 21$

Also, $(4x + 5)^2 - (x - 2)^2$ is a difference of squares, so:

$A = [(4x + 5) - (x - 2)][(4x + 5) + (x - 2)]$

$= (4x + 5 - x + 2)(4x + 5 + x - 2)$

$= (3x + 7)(5x + 3)$

A simplified expression for the area is $15x^2 + 44x + 21 = (3x + 7)(5x + 3)$.

b) The area of the shaded region will be the area of the large circle minus the area of the small circle.

$A = \pi(3x + 2)^2 - \pi(x + 1)^2$

$= \pi[(3x + 2)^2 - (x + 1)^2]$

$= \pi[(3x + 2)(3x + 2) - (x + 1)(x + 1)]$

$= \pi[9x^2 + 12x + 4 - (x^2 + 2x + 1)]$

$= \pi(9x^2 + 12x + 4 - x^2 - 2x - 1)$

$= \pi(8x^2 + 10x + 3)$

$= 8\pi x^2 + 10\pi x + 3\pi$

Also, $\pi(3x + 2)^2 - \pi(x + 1)^2$ is a difference of squares, so:

$A = \pi(3x + 2)^2 - \pi(x + 1)^2$

$A = \pi[(3x + 2)^2 - (x + 1)^2]$

$A = \pi[(3x + 2) - (x + 1)][(3x + 2) + (x + 1)]$

$A = \pi(3x + 2 - x - 1)(3x + 2 + x + 1)$

$A = \pi(2x + 1)(4x + 3)$

A simplified expression for the area is $8\pi x^2 + 10\pi x + 3\pi = \pi(2x + 1)(4x + 3)$.

15. Answers may vary, e.g.,

a)

Definition: A trinomial that has identical binomial factors	Characteristics: The first and last terms are perfect squares. The product of their square roots is doubled to get the number in the middle term.
Perfect Square	
Examples: $x^2 + 6x + 9$, $4y^2 - 8yz + z^2$	Non-examples: $x^2 - 6x - 9$, $8y^2 - 8yz + z^2$

b)

Definition: A binomial that has similar binomial factors; one contains a + sign, and the other contains a − sign	Characteristics: Both terms are perfect squares, separated by a subtraction sign.
Difference of Squares	
Examples: $x^2 - 36$, $4c^2 - 9d^2$	Non-examples: $x^2 - 35$, $4c^2 + 9d^2$

16. a) $(a + b)(a^2 - ab + b^2)$

$= a(a^2 - ab + b^2) + b(a^2 - ab + b^2)$

$= a(a^2) - ab(a) + a(b^2) + b(a^2) - ab(b) + b(b^2)$

$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$

$= a^3 + b^3$

b) Any sum of cubes such as $a^3 + b^3$ can be factored into $(a + b)(a^2 - ab + b^2)$.

c) To determine the a-value take the cube root of the first term, for the b-value take the cube root of the second term.

i) $a = x$, $b = 2$

$(a + b)(a^2 - ab + b^2) = (x + 2)(x^2 - 2x + 2^2)$

$= (x + 2)(x^2 - 2x + 4)$

ii) $a = x$, $b = 3$

$(a + b)(a^2 - ab + b^2) = (x + 3)(x^2 - 3x + 3^2)$

$= (x + 3)(x^2 - 3x + 9)$

iii) $a = 2x$, $b = 1$

$(a + b)(a^2 - ab + b^2) = (2x + 1)[(2x)^2 - 1(2x) + 1^2]$

$= (2x + 1)(4x^2 - 2x + 1)$

iv) $a = 3x, b = 2$

$$(a+b)(a^2-ab+b^2) = (3x+2)[(3x)^2-2(3x)+2^2]$$
$$= (3x+2)(9x^2-6x+4)$$

17. a) $(a-b)(a^2+ab+b^2)$

$$= a(a^2+ab+b^2)-b(a^2+ab+b^2)$$
$$= a(a^2)+ab(a)+a(b^2)-b(a^2)-ab(b)-b(b^2)$$
$$= a^3+a^2b+ab^2-a^2b-ab^2-b^3$$
$$= a^3-b^3$$

b) Any difference of cubes such as $a^3 - b^3$ can be factored into $(a-b)(a^2+ab+b^2)$.

c) To determine the a-value take the cube root of the first term, for the b-value take the cube root of the second term.

i) $a = x, b = 3$

$$(a-b)(a^2+ab+b^2) = (x-3)(x^2+3x+3^2)$$
$$= (x-3)(x^2+3x+9)$$

ii) $a = x, b = 4$

$$(a-b)(a^2+ab+b^2) = (x-4)(x^2+4x+4^2)$$
$$= (x-4)(x^2+4x+16)$$

iii) $a = 2x, b = 5$

$$(a-b)(a^2+ab+b^2) = (2x-5)[(2x)^2+2x(5)+5^2]$$
$$= (2x-5)(4x^2+10x+25)$$

iv) $a = 4x, b = 3$

$$(a-b)(a^2+ab+b^2) = (4x-3)[(4x)^2+4x(3)+(3)^2]$$
$$= (4x-3)(16x^2+12x+9)$$

Curious Math, page 232

1. starting number: 4
calculation: $4 \times 4 \times 4 \times 4 + 4$
result: 260
The result is a composite number since $260 = 26 \times 10$.

2. Yes, it appears to be true. We can verify this result with several different numbers, and the larger the number, the more likely it is to be composite. So, from preliminary observations, Jamie appears to be correct.

3. $n \times n \times n \times n + 4 = n^4 + 4$
Jamie's expression can be represented by $n^4 + 4$.

4. The new expression is $n^4 + 4 + 4n^2 - 4n^2$.

5. Begin by factoring the first three terms as a trinomial. Then notice the expression is a difference of squares.

$$n^4+4+4n^2-4n^2 = (n^4+4n^2+4)-4n^2$$
$$= (n^2+2)(n^2+2)-4n^2$$
$$= (n^2+2)^2-4n^2$$
$$= (n^2+2+2n)(n^2+2-2n)$$

6. Since there are two factors this expression must represent a composite number. If the number were prime its expression would not be factorable. Therefore Jamie is correct.

Lesson 4.6: Reasoning about Factoring Polynomials, page 236

1. a) Since there are three terms the polynomial is a trinomial. It is possible to divide out a common factor, since each term has a common monomial.

b) Since there are three terms the polynomial is a trinomial where $a \neq 1$. Decomposition can be used to factor this expression.

c) This polynomial has four or more terms. To factor try a grouping strategy.

d) Since there are two terms, this polynomial is a binomial. Use the difference of squares pattern, since both terms are perfect squares.

e) Since there are three terms, the polynomial is a trinomial where $a \neq 1$. Since each term is a multiple of 3 begin by common factoring. Then use the sum/product pattern to factor the remaining trinomial which now has $a = 1$.

f) Since there are three terms, the polynomial is a trinomial where $a = 1$. Use the sum/product pattern since $a = 1$.

2. a) Divide out the common factor of $2xy$. The expression cannot be factored further.

$$6xy+12x^2y^2-4x^3y^3$$
$$= 2xy(3+6xy-2x^2y^2)$$

The factored expression is $2xy(3 + 6xy - 2x^2y^2)$.

b) Two numbers whose product is $20 \times (-3) = -60$ and whose sum is 11 are 15 and -4.

$$20x^2+11x-3 = 20x^2+15x-4x-3$$
$$= 5x(4x+3)-1(4x+3)$$
$$= (5x-1)(4x+3)$$

c) Use a grouping strategy.

$$3x^2+3ax-2x-2a = 3x(x+a)-2(x+a)$$
$$= (3x-2)(x+a)$$

d) The square root of the first term is $7y$ and the square root of the second term is 3. The factored expression is $(7y - 3)(7y + 3)$.

e) First divide out the common factor of 3. Two numbers whose product is -30 and whose sum is -1 are 5 and -6.

$$3x^2-3x-90 = 3(x^2-x-30)$$
$$= 3(x+5)(x-6)$$

The factored expression is $3(x + 5)(x - 6)$.

f) Two numbers whose product is 42 and whose sum is -13 are -6 and -7.

$$x^2-13x+42 = (x-6)(x-7)$$

3. Answers may vary, e.g.,

a) i) $a = 1^2$, $c = (-3)^2$, and $b = 2a = 2(-3) = -6$
A possible expression is $x^2 - 6x + 9$.

ii) Any numbers can be chosen for b and c except 0. A possible expression is $x^2 + 11x - 12$.

iii) Each term must be squared and separated by a minus sign. A possible expression is $4x^2 - 36y^2$.

iv) Any number can be chosen for b and c except 0, and the a-value cannot equal 1. A possible expression is $10x^2 - x - 2$.

v) $4m^2 + 8$; This expression has a common factor of 4, a monomial.

vi) $3ab + 6a - 4b - 8$; This expression has a common factor of $3a - 4$, a binomial.

b) i) Using the perfect square pattern,
$x^2 - 6x + 9 = (x - 3)^2$

ii) Using the sum/product pattern,
$x^2 + 11x - 12 = (x - 1)(x + 12)$

iii) Using the difference of squares pattern,
$4x^2 - 36y^2 = (2x - 6y)(2x + 6y)$

iv) Using the sum/product pattern,
$10x^2 - x - 2 = (2x - 1)(5x + 2)$

v) Dividing out the GCF, $4m^2 + 8 = 4(m^2 + 2)$

vi) Using a grouping strategy,

$$3ab+6a-4b-8 = b(3a-4)+2(3a-4)$$
$$= (3a-4)(b+2)$$

4. a) $-10a^3 + 15a^2 = -5a^2(2a - 3)$
Therefore, ◆ = 2 and ■ = 3.
b) $7 \times ■ = 63$
$■ = 9$
$-◆ = 7 - ■$
$-◆ = = 7 - 9$
$-◆ = = -2$
Therefore, ◆ = 2 and ■ = 9.
c) $■^2 = 25$, so ■ = 5 or ■ = –5. Because there are only two terms on the left hand side of the equation, the left hand side must be a difference of squares (or it wouldn't be factorable like on the right hand side). So, it must be that ● = 7 and ◆ = 49.
d) $6x^2 = 2x \times ◆x$
$◆ = 3$
$-10 = -2 \times ■$
$■ = 5$
$● = -2 \times 2 + 3 \times 5$
$● = -4 + 15$
$● = 11$
Therefore, ● = 11; ■ = 5, and ◆ = 3.
5. $3x^2 + 11x + 10$
Since 5 and 6 add to 11 and have a product of 30, they will be used to factor this trinomial.
$3x^2 + 11x + 10 = 3x^2 + 6x + 5x + 10$
$= 3x(x + 2) + 5(x + 2)$
$= (3x + 5)(x + 2)$
Therefore, ▲ = 3, ● = 5, ■ = 1 and ◆ = 2 is one possible combination.
6. a) Since 16 and 25 are perfect squares and they are separated by a subtraction sign, this polynomial can be factored as a difference of squares.
$16x^2 - 25 = (4x - 5)(4x + 5)$
b) Each term has a common $-3b^2$. This will be factored out.
$-6b^2a - 9b^3 + 15b^2 = -3b^2(2a + 3b - 5)$
c) This is a trinomial where $a = 1$, the sum/product pattern can be used.
$-7 - 5 = -12$ and $-7 \times -5 = 35$
The factored expression is $(c - 7)(c - 5)$.
d) Since 49 and 1 are perfect squares and $14 = (2)(1)(7)$, the trinomial can be factored as a perfect square. The square root of the first term is $7d$ and the square root of the last term is 1. Since the middle term is positive the two terms will be separated by a plus sign, $(7d + 1)^2$.
e) This polynomial is a trinomial where $a \neq 1$, so factor using decomposition.
$12x^2 + 4x - 21 = 12x^2 + 18x - 14x - 21$
$= 6x(2x + 3) - 7(2x + 3)$
$= (6x - 7)(2x + 3)$
f) Since there are four terms, factor by grouping.
$2wz + 6w - 5z - 15 = 2w(z + 3) - 5(z + 3)$
$= (2w - 5)(z + 3)$

7. a) Factor using decomposition:
$10x^2 + 3x - 1 = 10x^2 - 2x + 5x - 1$
$= 2x(5x - 1) + 1(5x - 1)$
$= (5x - 1)(2x + 1)$
b) Factor using difference of squares:
$144a^4 - 121 = (12a^2 - 11)(12a^2 + 11)$
c) Factor by grouping:
$24ac - 8c + 21a - 7 = 8c(3a - 1) + 7(3a - 1)$
$= (8c + 7)(3a - 1)$
d) Use common factoring and the sum/product pattern:
$x^3 - 11x^2 + 18x = x(x^2 - 11x + 18)$
$= x(x - 9)(x - 2)$
e) Use common factoring and perfect-square trinomial factoring:
$18x^2 + 60x + 50 = 2(9x^2 + 30x + 25)$
$= 2(3x + 5)^2$
f) Use common factoring and differences of squares:
$x^2y - 4y = y(x^2 - 4)$
$= y(x - 2)(x + 2)$
8. a) Use common factoring and the sum/product pattern:
$2s^2 + 4s - 6 = 2(s^2 + 2s - 3)$
$= 2(s - 1)(s + 3)$
b) Use common factoring and the sum/product pattern:
$14 - 5w - w^2 = -(w^2 + 5w - 14)$
$= -(7 + w)(w - 2)$ or $(7 + w)(2 - w)$
c) Use decomposition and differences of squares:
$z^4 - 13z^2 + 36 = z^4 - 4z^2 - 9z^2 + 36$
$= z^2(z^2 - 4) - 9(z^2 - 4)$
$= (z^2 - 9)(z^2 - 4)$
$= (z - 3)(z + 3)(z - 2)(z + 2)$
d) Use common factoring and difference of squares:
$8s^2 - 50r^2 = 2(4s^2 - 25r^2)$
$= 2(2s - 5r)(2s + 5r)$
e) Use perfect-square trinomial factoring:
$49g^2 - 84g + 36 = (6 - 7g)^2$
f) Use perfect-square trinomial factoring and differences of squares:
$x^2 + 10x + 25 - 16y^2 = (x^2 + 10x + 25) - 16y^2$
$= (x + 5)^2 - 16y^2$
$= (x + 5 - 4y)(x + 5 + 4y)$
9. a) This expression is not fully factored since $x^2 - 1$ can be factored as $(x - 1)(x + 1)$.
b) This expression is not fully factored since $x^2 - 9x + 20$ can be factored as $(x - 5)(x - 4)$.
c) This expression is not fully factored as the coefficients of the $(5x + 2y)$ terms can be grouped to get $(5x + 2y)(3x - 1)$.
d) This expression is not fully factored since $16a^4 - b^4$ can be factored as $(2a - b)(2a + b)(4a^2 + b^2)$.
10. a) Use a grouping strategy.
$xy - ty + xs - ts = y(x - t) + s(x - t)$
$= (x - t)(y + s)$
b) Since 25 and 16 are perfect squares this polynomial is a difference of squares.
$-25x^2 + 16y^2 = 16y^2 - 25x^2$
$= (4y - 5x)(4y + 5x)$
c) Use the perfect square pattern, then differences of squares.
$a^4 - 13a^2 + 36 = (a^2 - 4)(a^2 - 9)$
$= (a - 2)(a + 2)(a - 3)(a + 3)$

d) Use the perfect squares pattern, then difference of squares.

$$\begin{aligned} x^2 - y^2 - 2y - 1 &= x^2 - (y^2 + 2y + 1) \\ &= x^2 - (y + 1)^2 \\ &= (x - y - 1)(x + y + 1) \end{aligned}$$

e) Use decomposition.

$$\begin{aligned} &2(a + b)^2 + 5(a + b) + 3 \\ &= 2(a + b)^2 + 2(a + b) + 3(a + b) + 3 \\ &= 2(a + b)([a + b] + 1) + 3([a + b] + 1) \\ &= [2(a + b) + 3](a + b + 1) \\ &= (2a + 2b + 3)(a + b + 1) \end{aligned}$$

f) Divide by the common factor, then use decomposition.

$$\begin{aligned} 6x^3 - 63x - 13x^2 &= x(6x^2 - 63 - 13x) \\ &= x(6x^2 - 13x - 63) \\ &= x[3x(2x - 9) + 7(2x - 9)] \\ &= x(3x + 7)(2x - 9) \end{aligned}$$

11. a) Use decomposition.

$$\begin{aligned} A &= 8x^2 + 18x + 7 \\ &= 8x^2 + 4x + 14x + 7 \\ &= 4x(2x + 1) + 7(2x + 1) \\ &= (4x + 7)(2x + 1) \end{aligned}$$

The length is $4x + 7$ and the width is $2x + 1$, or vice versa.

b)

$$\begin{aligned} \text{length} &= 4x + 7 \\ &= 4(3) + 7 \\ &= 12 + 7 \\ &= 19; \\ \text{width} &= 2x + 1 \\ &= 2(3) + 1 \\ &= 6 + 1 \\ &= 7; \\ \text{area} &= l \times w \\ &= 19 \times 7 \\ &= 133 \end{aligned}$$

The length is 19 cm and the width is 7 cm, or vice versa. The area is 133 cm^2.

12. a)

$$\begin{aligned} A &= \pi x^2 + 10\pi x + 25\pi \\ &= \pi(x^2 + 10x + 25) \\ &= \pi(x + 5)^2 \end{aligned}$$

Since $A = \pi r^2$, the radius is $x + 5$.

b)

$$\begin{aligned} r &= x + 5 \\ &= 10 + 5 \\ &= 15; \\ A &= \pi r^2 \\ &= \pi(x + 5)^2 \\ &= 225\pi \end{aligned}$$

The radius is 15 cm and the area is 225π cm^2.

13. a) Use decomposition.

$$\begin{aligned} V &= 2x^3 + 14x^2 + 24x \\ V &= 2x(x^2 + 7x + 12) \\ V &= 2x(x + 4)(x + 3) \end{aligned}$$

The three dimensions are $2x$, $x + 4$, and $x + 3$.

b)

$$\begin{aligned} 2x &= 2(5) = 10; \\ x + 4 &= 5 + 4 = 9; \\ x + 3 &= 5 + 3 = 8; \\ \text{volume} &= 10 \times 5 \times 8 \\ &= 720 \end{aligned}$$

The dimensions are 10 cm, 9 cm, and 8 cm. The volume is 720 cm^3.

14. a) No. After combining like terms, the expression is $9x^2y - 3xy$. The expression can then be factored as $3xy(3x - 1)$, thus $(x + y)$ is not a factor.

b) Yes. Use a grouping strategy, then divide out the common factor, then use difference of squares.

$$\begin{aligned} x^5 - x^3y^2 + x^3 - xy^2 &= x^3(x^2 - y^2) + x(x^2 - y^2) \\ &= (x^3 + x)(x^2 - y^2) \\ &= (x^3 + x)(x^2 - y^2) \\ &= x(x^2 + 1)(x^2 - y^2) \\ &= x(x + y)(x - y)(x^2 + 1) \end{aligned}$$

Thus, $x + y$ is a factor.

c) No. Combine like terms, then divide out the common factor, then use the sum/product pattern.

$$\begin{aligned} x^2y + 6y - 9xy + x^2y + xy &= 2x^2y + 6y - 8xy \\ &= 2y(x^2 + 3 - 4x) \\ &= 2y(x^2 - 4x + 3) \\ &= 2y(x - 3)(x - 1) \end{aligned}$$

The expression can be factored as $2y(x - 3)(x - 1)$. Thus, $(x + y)$ is not a factor.

d) Yes. Use a grouping strategy, then use the sum/product strategy.

$$\begin{aligned} &x^3 + 5x^2 + 6x + x^2y + 5xy + 6y \\ &= x(x^2 + 5x + 6) + y(x^2 + 5x + 6) \\ &= (x + y)(x^2 + 5x + 6) \\ &= (x + y)(x + 3)(x + 2) \end{aligned}$$

The expression can be factored as $(x + y)(x + 3)(x + 2)$. Thus $x + y$ is a factor.

15. Answers may vary, e.g.,

Factoring a Polynomial

Factor out all common monomial factors.

↓

Check for a difference of squares:
$a^2 - b^2 = (a - b)(a + b)$

↓

Check for a perfect square:
$a^2 + 2ab + b^2 = (a + b)^2$ or
$a^2 - 2ab + b^2 = (a - b)^2$

↓

Check for the form:
$x^2 + bx + c = (x + r)(x + s)$, where $rs = c$ and $r + s = b$

↓

Check for the form:
$ax^2 + bx + c = (px + r)(qx + s)$, where $pq = a$, $rs = c$, and $ps + qr = b$

↓

If there are four or more terms, try a grouping strategy.

16. a) This is a difference of squares.

$$\frac{x^2}{9}-\frac{1}{4}=\left(\sqrt{\frac{x^2}{9}}-\sqrt{\frac{1}{4}}\right)\left(\sqrt{\frac{x^2}{9}}+\sqrt{\frac{1}{4}}\right)$$
$$=\left(\frac{x}{3}-\frac{1}{2}\right)\left(\frac{x}{3}+\frac{1}{2}\right)$$

b) This is a difference of squares.

$100-(a-5)^2 = [10-(a-5)][10+(a-5)]$
$= (10-a+5)(10+a-5)$
or $(15-a)(5+a)$

c) Since this is an addition of squares, no factoring is possible.

d) This is a difference of squares.

$$\frac{25a^2}{64}-\frac{9b^2}{49}=\left(\sqrt{\frac{25a^2}{64}}-\sqrt{\frac{9b^2}{49}}\right)\left(\sqrt{\frac{25a^2}{64}}+\sqrt{\frac{9b^2}{49}}\right)$$
$$=\left(\frac{5a}{8}-\frac{3b}{7}\right)\left(\frac{5a}{8}+\frac{3b}{7}\right)$$

e) This is a difference of squares:

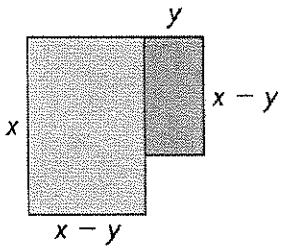
$(x+3)^2-(y-3)^2 = (x+3)-(y-3)[(x+3)+(y-3)]$
$= (x+3+y-3)(x+3-y+3)$
$= (x+y)(x-y+6)$

f) This trinomial is a perfect-square trinomial.

$4(c-5)^4+12(c-5)^2+9=[2(c-5)^2+3)]^2$

This expression can be factored to $[2(c-5)^2+3]^2$ or $(2c^2-20c+53)^2$.

17. Answers may vary, e.g., I divided the remaining area into two parts as follows:



The large rectangle has an area of $x(x-y)$ and the small rectangle has an area of $y(x-y)$. Adding these gives $(x+y)(x-y)$. Since the combined area is equal to the area of the large square minus the area of the small square, $x^2-y^2=(x+y)(x-y)$.

Chapter Review, page 240

1. a) Add like terms to get the expression $6x-9$. This expression is equal to the area of a rectangle of length $(2x-3)$ and width 3. So, $6x-9=3(2x-3)$.

b) Add like terms to get the expression $2x^2+8x$. This expression is equal to the area of a rectangle of length $(x+4)$ and width $2x$. Therefore, $2x^2+8x=2x(x+4)$.

2. a) Divide out the GCF. $20x^2-4x=4x(5x-1)$

b) Divide out the GCF. $3n^2-6n+15=3(n^2-2n+5)$

c) Divide out the GCF.
$-2x^3+6x^2+4x=-2x(x^2-3x-2)$

d) Divide out the GCF.
$6a(3-7a)-5(3-7a)=(3-7a)(6a-5)$

3. a) Answers may vary, e.g., possible dimensions are factors of the given area: $16x^2-24=8(2x^2-3)$
Therefore, the rectangle could have a length of 8 and a width of $2x^2-3$ or vice versa.

b) Yes, $16x^2-24$ can be factored in other ways, such as:
$16x^2-24 = 2(8x^2-12)$ or
$= 4(4x^2-6)$
Therefore, the rectangle could have dimensions of 2 by $8x^2-12$ or 4 by $4x^2-6$ as well.

4. a) Answers may vary, e.g., choose any three non-factorable polynomials and multiply them by $4x^3y$ to create polynomials with $4x^3y$ as their GCF.

$4x^3y(1+3xz)$
(1) $= 4x^3y+12x^4yz$;
$4x^3y(2x^2y-z+10y^2)$
(2) $= 8x^5y^2-4x^3yz+40x^3y^3$;
$4x^3y(4z+3)$
(3) $= 16x^3yz+12x^3y$

b) $4x^3y$ is the greatest common factor for each polynomial in part a). Therefore, factoring this term out of each polynomial leaves the factored form that was used to generate the polynomials in the first place.
Therefore
(1) $4x^3y+12x^4yz=4x^3y(1+3xz)$;
(2) $8x^5y^2-4x^3yz+40x^3y^3=4x^3y(2x^2y-z+10y^2)$;
(3) $16x^3yz+12x^3y=4x^3y(4z+3)$

5. a) Adding like terms shows that this rectangle has an area defined by the expression x^2+3x-4. The rectangle has a length of $x-1$ and a width of $x+4$,so these are the factors.
$x^2+x-4=(x+4)(x-1)$.

b) Summing like terms reveals that this rectangle has an area defined by the expression $2x^2+5x-3$. The rectangle has a length of $2x-1$ and a width of $x+3$, so these are the factors.
$2x^2+5x-3=(x+3)(2x-1)$

6. a) Determine two numbers with a sum of 16 and a product of 63. The product of the numbers is positive, so they must be either both positive or both negative. The sum of the two numbers is also positive, so they must both be positive. The numbers are 9 and 7, since $9\times7=63$ and $9+7=16$.
$x^2+16x+63=(x+9)(x+7)$

b) Draw different area diagrams until you find one that has an area of $x^2-7x+60$.

	x	-12
x	x^2	$-12x$
5	$5x$	-60

This rectangle has the correct area. It has a length of $(x-12)$, and a width of $(x+5)$; so, these are the factors.
$x^2-7x-60=(x-12)(x+5)$

c) Determine two numbers with a sum of 6 and a product of –27. The product of the numbers is negative, so they must be of opposite sign. The sum of the two numbers is positive, so the positive number must be of greater magnitude than the negative. The numbers are –3 and 9, since $-3 \times 9 = -27$ and $-3 + 9 = 6$. So, $x^2 + 6x - 27 = (x + 9)(x - 3)$

d) First, factor a common factor of 5 out of the expression: $5x^2 - 5x - 100 = 5(x^2 - x - 20)$. Determine two numbers with a sum of –1 and a product of –20. The numbers are –5 and 4, since $-5 \times 4 = -20$ and $-5 + 4 = -1$.

$5(x^2 - x - 20) = 5(x - 5)(x + 4)$

7. a) Determine two numbers with a sum of 7 and a product of 12. The numbers are 3 and 4, since $3 \times 4 = 12$ and $3 + 4 = 7$.

$y = x^2 + 7x + 12$

$y = (x + 3)(x + 4)$

b) Since the x-intercepts occur where $y = 0$, the factored form of the relation reveals that the x-intercepts have coordinates (–3, 0) and (–4, 0).

c) The x-coordinate of the vertex occurs halfway between the zeros by symmetry, at $x = -3.5$. Substitute this value into the relation in either form to determine the y-coordinate of the vertex.

$y = x^2 + 7x + 12$

$y = (-3.5)^2 + 7(-3.5) + 12$

$y = 12.25 - 24.5 + 12$

$y = -0.25$

Therefore, the vertex has coordinates (3.5, –0.25).

d) The parabola opens upward, since $a > 0$. Therefore, the minimum value occurs at $x = 3.5$, and has a value of $y = -0.25$.

8. a) Summing like terms reveals the expression for the area of this rectangle to be $x^2 + 18x + 80$. The length of the rectangle is $x + 10$ and the width is $x + 8$, so these are the factors of the expression.

$x^2 + 18x + 80 = (x + 10)(x + 8)$

b) Sum like terms to determine the area of the rectangle, and then use a decomposition strategy to determine the factors.

$$\begin{aligned} 10x^2 - 4x - 35x + 14 &= 2x(5x - 2) - 7(5x - 2) \\ &= (5x - 2)(2x - 7) \end{aligned}$$

The factors are $(5x - 2)$ and $(2x - 7)$.

9. Answers may vary, e.g., use a decomposition strategy.

a) Look for factors of the form $(px + r)(qx + s)$ with $rs = -4$, $pq = 15$, and $ps + qr = -4$.

b) Look for factors of the form $(px + r)(qx + s)$ with $rs = -2$, $pq = 20$, and $ps + qr = 3$.

c) Look for factors of the form $(px + r)(qx + s)$ with $rs = -16$, $pq = 7$, and $ps + qr = 6$.

d) Look for factors of the form $(px + r)(qx + s)$ with $rs = -10$, $pq = 20$, and $ps + qr = -17$.

10. a) Use a combination of reasoning and guess and test to determine the solution. 7 is prime, therefore $7x^2 - 19x - 6 = (7x + r)(x + s)$.

Then, $7s + r = -19$ and $rs = -6$. The factors of –6 are 1, –1, 2, –2, 3, –3, 6, –6, so these are possible values for r and s.

Try $r = 3$ and $s = -2$.

$$\begin{aligned} 7s + r &= 7(-2) + (3) \\ &= -14 + 3 \\ &= -11 \\ &\neq -19 \end{aligned}$$

These values are incorrect. Try $r = 2$ and $s = -3$.

$$\begin{aligned} 7s + r &= 7(-3) + (2) \\ &= -21 + 2 \\ &= -19 \end{aligned}$$

Therefore, these are the correct values.

$7x^2 - 19x - 6 = (7x + 2)(x - 3)$.

b) I looked for two numbers whose sum is 23 and whose product is 60. I know that both numbers must be positive. The numbers are 3 and 20. I used these numbers to decompose the middle term. I factored the first two terms and then the last two terms. I divided out the common factor of $a + 5$.

$$\begin{aligned} 4a^2 + 23a + 15 &= 4a^2 + 3a + 20a + 15 \\ &= a(4a + 3) + 5(4a + 3) \\ &= (a + 5)(4a + 3) \end{aligned}$$

c) Use a decomposition strategy to determine the solution. The factors are of the form $(px + r)(qx + s)$; therefore, $pq = 12$, $rs = 5$, and $ps + qr = -16$. Multiplying the first coefficient by the constant gives $(pq)(rs) = (ps)(qr) = 60$. 60 has factors ±1, ±2, ±3, ±4, ±5, ±6, ±10, ±12, ±15, ±20, ±30, and ±60. Since $ps + qr = -16$, $ps = -6$ and $qr = -10$.

$$\begin{aligned} 12x^2 - 16x + 5 &= 12x^2 - 6x - 10x + 5 \\ &= 6x(2x - 1) - 5(2x - 1) \\ &= (2x - 1)(6x - 5) \end{aligned}$$

d) Use a decomposition strategy to determine the solution. The factors are of the form $(px + r)(qx + s)$; therefore, $pq = 6$, $rs = -10y^2$, and $ps + qr = -11y$. Multiplying the first coefficient by the constant gives $(pq)(rs) = (ps)(qr) = -60y^2$. $-60y^2$ has factors $\pm y$, $\pm 2y$, $\pm 3y$, $\pm 4y$, $\pm 5y$, $\pm 6y$, $\pm 10y$, $\pm 12y$, $\pm 15y$, $\pm 20y$, $\pm 30y$, and $\pm 60y$. Since $ps + qr = -11y$, $ps = 4y$ and $qr = -15y$.

$$\begin{aligned} 6n^2 - 11ny - 10y^2 &= 6n^2 + 4ny - 15ny - 10y^2 \\ &= 2n(3n + 2y) - 5y(3n + 2y) \\ &= (2n - 5y)(3n + 2y) \end{aligned}$$

11. a) Use a guess and test strategy to factor the relation.

$P = -2n^2 + 120n - 1000$

$P = -2(n^2 - 60n + 500)$

The factored equation must be in the form $P = -2(n + r)(n + s)$, where $rs = 500$ and $r + s = -60$.

Try $r = -20$ and $s = -25$.

$-20 - 25 \neq -60$; so, these values are incorrect. Try $r = -50$ and $s = -10$. These values are correct, since $-50 - 10 = -60$. Thus, $P = -2(n - 50)(n - 10)$.

By inspection, $P = 0$ when $n = 10$ or $n = 50$. Therefore, Erica and Asif break even when they sell either 10 or 50 watches.

b) By the symmetry of quadratic relations, the maximum will occur for an n exactly halfway between the break even points. Thus, Erica and Asif earn the maximum profit when $n = 30$.

Substitute this value into the original equation to solve for P.

$P = -2n^2 + 120n - 1000$
$P = -2(30)^2 + 120(30) - 1000$
$P = -1800 + 3600 - 1000$
$P = 800$

Therefore, Eric and Asif generate a maximum profit of $800.

12. a) Summing like terms reveals this rectangle to have an area of $4x^2 + 12x + 9$. The tiles model this expression. The lengths of the rectangle are both $(2x + 3)$ The expression $4x^2 + 12x + 9$ has two equal factors of $2x + 3$.

$4x^2 + 12x + 9 = (2x + 3)^2$

b) Summing like terms reveals this rectangle to have an area of $64x^2 - 9$, which is the expression being modelled. The top left corner appears to be a square with side lengths of $8x$, which means that the unknown side length of the $-24x$ term must be -3, and the unknown side length of the $24x$ term must be 3. These predictions are correct, as $3 \times (-3) = 9$, the area of the bottom right corner. The side lengths are $(8x + 3)$ and $(8x - 3)$ so $64x^2 - 9 = (8x + 3)(8x - 3)$.

13. a) $144x^2$ and 25 are both perfect squares with square roots of $12x$ and 5 respectively, and 25 has a negative sign in front of it. This expression is a difference of squares. Therefore

$144x^2 - 25 = (12x - 5)(12x + 5)$

b) $36a^2$ and 1 are both positive perfect squares with square roots of $6a$ and 1 respectively. Also, $2 \times 6a \times 1 = 12a$, which is the other term in the trinomial. Therefore, this expression is a perfect-square trinomial: $36a^2 + 12a + 1 = (6a + 1)^2$

c) First, remove a common factor of $2x$:

$18x^5 - 512xy^2 = 2x(9x^4 - 256y^2)$.

$9x^4$ and $256y^2$ are both perfect squares with square roots of $3x^2$ and $16y$ respectively, and $256y^2$ has a negative sign in front of it. This expression is a difference of squares.

Therefore $18x^5 - 512xy^2 = 2x(3x^2 - 16y)(3x^2 + 16y)$.

d) $4(x - 2)^2$ and 25 are both positive perfect squares with square roots of $2(x - 2)$ and 5 respectively. Also, $-(2 \times 2(x - 2) \times 5) = -20(x - 2)$, which is the other term. Therefore this is a perfect-square trinomial.

$$\begin{aligned} 4(x-2)^2 + 20(x-2) + 25 &= [2(x-2) - 5]^2 \\ &= (2x - 4 - 5)^2 \\ &= (2x - 9)^2 \end{aligned}$$

e) $(x + 5)^2$ and y^2 are perfect squares, with square roots of $(x + 5)$ and y respectively. Also, y^2 has a negative sign in front of it. This expression is a difference of squares.

$$\begin{aligned} (x+5)^2 - y^2 &= [(x+5) - y][(x+5) + y] \\ &= (x - y + 5)(x + y + 5) \end{aligned}$$

f) x^2 and 9 are both positive perfect squares with square roots of x and 3 respectively. Also, $-(2 \times x \times 3) = -6x$, which is the middle term of the first three terms of this trinomial. Therefore, the first three terms form a perfect-square trinomial.

$x^2 - 6x + 9 - 4y^2 = (x - 3)^2 - 4y^2$

$(x - 3)^2$ and $4y^2$ are both perfect squares, with square roots of $(x - 3)$ and $2y$ respectively. Also, $4y^2$ has a negative sign in front of it.

This expression is a difference of squares.

$$\begin{aligned} x^2 - 6x + 9 - 4y^2 &= (x-3)^2 - 4y^2 \\ &= [(x-3) - 2y][(x-3) + 2y] \\ &= (x - 2y - 3)(x + 2y - 3) \end{aligned}$$

14. No. Answers may vary, e.g., $x^2 + 25$ cannot be factored because it has two terms and no common factors, and it is not a difference of squares.

15. Expanding an algebraic expression is the reverse of factoring it. Answers may vary, e.g., expanding $(x + 2)(x - 4)$ gives $x^2 - 2x - 8$, while factoring $x^2 - 2x - 8$ gives $(x + 2)(x - 4)$.

16. a) This expression has three terms, none of which are perfect squares. The first coefficient is prime; therefore, the factors are of the form $(7x + r)(x + s)$. Determine r and s such that their product is -8 and $r + 7s = -26$. Letting $r = 2$ and $s = -4$ satisfies these conditions.

$7x^2 - 26x - 8 = (7x + 2)(x - 4)$

b) Both $64a^6$ and 25 are perfect squares with square roots of $8a^3$ and 5 respectively, and 25 has a negative sign in front of it. Therefore, this is a difference of squares.

$64a^6 - 25 = (8a^3 - 5)(8a^3 + 5)$

c) Factor using a grouping strategy.

$$\begin{aligned} 18ac - 12a - 15c + 10 &= 6a(3c - 2) - 5(3c - 2) \\ &= (6a - 5)(3c - 2) \end{aligned}$$

d) First, factor out a common factor of $4y$:

$4x^2y - 44xy + 72y = 4y(x^2 - 11x + 18)$.

To factor the remaining trinomial, determine two numbers whose product is 18 and whose sum is -11. These numbers are -9 and -2.

$4x^2y - 44xy + 72y = 4y(x - 9)(x - 2)$

e) This expression has three terms, none of which are perfect squares. Use a decomposition strategy to factor. The first coefficient and constant have a product of 900. Determine two factors of 900 whose sum is 61. The two factors are 36 and 25.

$$\begin{aligned} 20x^2 + 61x + 45 &= 20x^2 + 25x + 36x + 45 \\ &= 5x(4x + 5) + 9(4x + 5) \\ &= (5x + 9)(4x + 5) \end{aligned}$$

f) Let $x = z^2$. This trinomial is now $x^2 - 13x + 40$. This trinomial has three terms, and the first coefficient is 1. Solutions are of the form $(x + r)(x + s)$. Determine values for r and s such that $rs = 40$ and $r + s = -13$. Therefore, $r = -8$ and $s = -5$.

$$\begin{aligned} z^4 - 13z^2 + 40 &= x^2 - 13x + 40 \\ &= (x - 8)(x - 5) \\ &= (z^2 - 8)(z^2 - 5) \end{aligned}$$

17. a) This expression is a trinomial with no terms that are perfect squares. Factors are of the form $(px + r)(qx + s)$. Determine p, q, r, and s such that $pq = 2$, $rs = -5$, and $ps + qr = 3$. 2 is a prime number, so let $p = 1$ and $q = 2$. Then, $s + 2r = 3$. This equation is satisfied if $s = 5$ and $r = -1$.
$2s^2 + 3s - 5 = (s - 1)(2s + 5)$
b) This expression is a trinomial with no terms that are perfect squares. As w^2 has a coefficient of -1, solutions must be of the form $(r + w)(s - w)$. Determine r and s such that $rs = 15$, and $s - r = -2$. Therefore, the values must be $r = 5$ and $s = 3$.
$15 - 2w - w^2 = (5 + w)(3 - w)$
c) Let $x = z^2$. The expression becomes $x^2 - 4x - 32$. This expression has three terms, none of which are perfect squares. The first coefficient is 1, so factors will be of the form $(x + r)(x + s)$. Determine the values of r and s such that $rs = -32$ and $r + s = -4$. The values are $r = 4$ and $s = -8$.

$$\begin{aligned} z^4 - 4z^2 - 32 &= x^2 - 4x - 32 \\ &= (x + 4)(x - 8) \end{aligned}$$

Substitute $x = z^2$ back into the equation to get $(z^2 + 4)(z^2 - 8)$
d) This expression contains two terms, both of which are perfect squares and one of which is subtracted from the other so it is a difference of squares. The square root of $16s^2$ is $4s$ and the square root of $121r^2$ is $11r$.
$16s^2 - 121r^2 = (4s - 11r)(4s + 11r)$
e) 9 and $25g^2$ are perfect squares with square roots of 3 and $5g$ respectively. Also, $-(2 \times 3 \times 5g) = -30g$, which is the other term in the trinomial. Therefore, this is a perfect-square trinomial.
$9 - 30g + 25g^2 = (3 - 5g)^2$
f) The first term and constant are both perfect squares in this expression, with square roots of x and 8 respectively. Also, $2 \times x \times 8 = 16x$, which is the middle of the first 3 terms. Therefore, the first three terms form a perfect-square trinomial.
$x^2 + 16x + 64 - 25y^2 = (x + 8)^2 - 25y^2$
Both of the terms in this expression are perfect squares separated by a negative sign. Therefore, this expression is a difference of squares, with square roots of $x + 8$ and $5y$.

$$\begin{aligned} (x + 8)^2 - 25y^2 &= [(x + 8) - 5y][(x + 8) + 5y] \\ &= (x - 5y + 8)(x + 5y + 8) \end{aligned}$$

18. a) Factor the given expression into three factors to determine the dimensions of the box. Use a grouping strategy.

$$\begin{aligned} V &= 18x^3 - 2x + 45x^2 - 5 \\ V &= 2x(9x^2 - 1) + 5(9x^2 - 1) \\ V &= (2x + 5)(9x^2 - 1) \end{aligned}$$

Observe that the second factor has two terms that are perfect squares, and which are separated by a negative sign. This factor is a difference of squares. The square roots of each term are $3x$ and 1.

$$\begin{aligned} V &= (2x + 5)(9x^2 - 1) \\ V &= (2x + 5)(3x - 1)(3x + 1) \end{aligned}$$

These three factors represent the length, width, and height of the box. The box is $(2x + 5)$ by $(3x - 1)$ by $(3x + 1)$.

b)

$$\begin{aligned} 2x + 5 &= 2(2) + 5 \\ &= 9; \\ 3x - 1 &= 3(2) - 1 \\ &= 5; \\ 3x + 1 &= 3(2) + 1 \\ &= 7; \\ V &= (2x + 5)(3x - 1)(3x + 1) \\ V &= (9)(5)(7) \\ V &= 315 \end{aligned}$$

If $x = 2$ cm, the box will have dimensions of 9 cm by 5 cm by 7 cm, and will have a volume of 315 cm^3.
19. a) This expression has a first coefficient of 1, and a constant of 24. Determine two factors of $1 \times 24 = 24$ that have a sum of -10. The factors are -4 and -6.

$$\begin{aligned} y &= x^2 - 10x + 24 \\ y &= (x - 4)(x - 6) \end{aligned}$$

By inspection, the zeros occur at $x = 4$ and $x = 6$ and by symmetry, the x-coordinate of the vertex is halfway between these zeros, so $x = 5$. Substitute this value into the original equation and solve for y.

$$\begin{aligned} y &= x^2 - 10x + 24 \\ y &= 5^2 - 10(5) + 24 \\ y &= 25 - 50 + 24 \\ y &= -1 \end{aligned}$$

Therefore, the vertex has coordinates $(5, -1)$.
b) First divide out a common factor of 2. Now the first coefficient and constant are both perfect squares, with square roots of 1 and 6 respectively.
$-(2 \times 1 \times 6) = -12$, which is the coefficient of x. This is a perfect- square trinomial, with two equal factors of $(x - 6)$.

$$\begin{aligned} y &= 2x^2 - 24x + 72 \\ y &= 2(x^2 - 12x + 36) \\ y &= 2(x - 6)^2 \end{aligned}$$

There is only one zero, which is also the vertex. By inspection, this zero occurs at $x = 6$. Therefore, the vertex has coordinates $(6, 0)$.
c) First divide out a common factor of -5. Now the first coefficient and constant are both perfect squares that are separated by a negative sign, with square roots of 1 and 100. This is a difference of squares.

$$\begin{aligned} y &= -5x^2 + 500 \\ y &= -5(x^2 - 100) \\ y &= -5(x - 10)(x + 10) \end{aligned}$$

By inspection, the zeros occur at $x = -10$ and $x = 10$ and by symmetry, the x-coordinate of the vertex is halfway between these zeros, so $x = 0$. Substitute this value into the original equation and solve for y.

$$\begin{aligned} y &= -5x^2 + 500 \\ y &= -5(0)^2 + 500 \\ y &= 500 \end{aligned}$$

Therefore, the vertex has coordinates $(0, 500)$.

d) This expression has a first coefficient of 2, and a constant of –4. Determine two factors of $2 \times -4 = -8$ that have a sum of –7. The factors are –8 and 1.

$y = 2x^2 - 7x - 4$
$y = 2x^2 - 8x + x - 4$
$y = 2x(x - 4) + (x - 4)$
$y = (2x + 1)(x - 4)$

By inspection, the zeros $x = -0.5$ and $x = 4$ and by symmetry, the x-coordinate of the vertex is halfway between these zeros, so $x = 1.75$. Substitute this value into the original equation and solve for y.

$y = 2x^2 - 7x - 4$
$y = 2(1.75)^2 - 7(1.75) - 4$
$y = 6.125 - 12.25 - 4$
$y = -10.125$

Therefore, the vertex has coordinates (1.75, –10.125).

e) First divide out the common factor of $4x$.

$y = 4x^2 + 16x$
$y = 4x(x + 4)$

By inspection, the zeros occur at $x = -4$ and $x = 0$ and by symmetry, the x-coordinate of the vertex is halfway between these zeros, so $x = -2$. Substitute this value into the original equation to solve for y.

$y = 4x^2 + 16x$
$y = 4(-2)^2 + 16(-2)$
$y = 16 - 32$
$y = -16$

Therefore, the vertex has coordinates (–2, –16).

f) The first coefficient and constant are both perfect squares, with square roots of 1 and 5 respectively. $2 \times 1 \times 5 = 10$, which is the coefficient of x. This is a perfect-square trinomial, with two equal factors of $(x + 5)$.

$y = (x + 5)^2$

There is only one zero, which is also the vertex. By inspection, this zero occurs at $x = -5$. Therefore, the vertex has coordinates (–5, 0).

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1. a) From the equation:
(1) $7■ = 56$ (2) $-■ + 7 = -◆$
Solve relation 1 for ■.

$7■ = 56$
$■ = 8$

Substitute ■ = 8 into relation 2 to solve for ◆.

$-■ + 7 = -◆$
$-(8) + 7 = -◆$
$◆ = 1$

Therefore, ■ = 8 and ◆ = 1.

b) From the equation:
(1) $■^2 = 16$ (2) $-3■ + ●■ = 0$ (3) $3● = ◆$
Solve relation 1 for ■.

$■^2 = 16$
$■ = 4$ or -4

Substitute ■ = 4 into relation 2 to solve for ●.

$-3■ + ●■ = 0$
$-3(4) + ●(4) = 0$
$-12 + 4● = 0$
$4● = 12$
$● = 3$

Substitute ■ = –4 into relation 2 to solve for ●.

$-3■ + ●■ = 0$
$-3(-4) + ●(-4) = 0$
$12 - 4● = 0$
$4● = 12$
$● = 3$

Whether ■ = –4 or +4, ● = 3. Substitute this value into relation 3 to solve for ◆.

$3● = ◆$
$3(3) = ◆$
$◆ = 9$

Therefore ■ = 4 or –4, ● = 3, and ◆ = 9.

c) From the equation:
(1) $4◆ = 12$ (2) $■◆ + 20 = ●$ (3) $5 = 5■$
Solve relation 1 for ◆.

$4◆ = 12$
$◆ = 3$

Solve relation 3 for ■.

$5 = 5■$
$■ = 1$

Substitute these values into relation 2 to solve for ●.

$■◆ + 20 = ●$
$(1)(3) + 20 = ●$
$● = 23$

Therefore, ◆ = 3, ■ = 1, and ● = 23.

d) From the equation:
(1) $■^2 = 25$ (2) $● = 2■◆$ (3) $◆^2 = 49$
Solve relation 1 for r ■.

$■^2 = 25$
$■ = -5$ or 5

Solve relation 3 for ◆.

$◆^2 = 49$
$◆ = -7$ or 7

Substitute these values into relation 2 to solve for ●.
For ■ and ◆ of the same sign:

$● = 2■◆$
$● = 2(5)(7)$
$● = 70$

For ■ and ◆ of opposite sign:

$● = 2■◆$
$● = 2(-5)(7)$
$● = -70$

Four possibilities exist: ■ = –5, ◆ = –7, and ● = 70; ■ = –5, ◆ = 7, and ● = –70; ■ = 5, ◆ = –7, and ● = –70; and ■ = 5, ◆ = 7, and ● = 70.

2. a) Adding like terms shows this rectangle has an area of $4x^2 - 10x + 6$. It is $(2x - 3)$ units long and $(2x - 2)$ wide, so these are its factors.

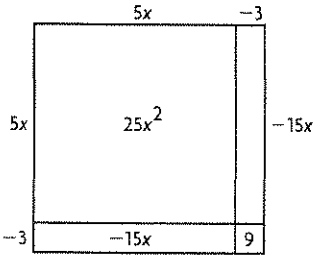
$4x^2 - 10x + 6 = (2x - 3)(2x - 2)$

b) Adding like terms shows this rectangle has an area of $2x^2 + 7x - 4$. The first row has a common factor of x, meaning that the length of the entire rectangle is $(2x - 1)$, and that the width of the top half is x. Factoring $2x$ out of the bottom left corner gives the lower half a width of 4. The same is true for the bottom right corner when factoring out –1, confirming that this width is correct. Adding the widths of the top and bottom half gives $(x + 4)$. Then, the rectangle has dimensions of $(2x - 1)$ by $(x + 4)$. So the trinomial can be factored as $2x^2 + 7x - 4 = (2x - 1)(x + 4)$.

3. a) Divide out the GCF. $20x^5 - 30x^3 = 10x^3(2x^2 - 3)$
b) Divide out the CGF.
$-8yc^3 + 4y^2c - 6yc = -2yc(4c^2 - 2y + 3)$
c) Divide out the GCF.
$2a(3b + 5) + 7(3b + 5) = (2a + 7)(3b + 5)$
d) Use a grouping strategy.

$$\begin{aligned} 2st + 6s + 5t + 15 &= 2s(t + 3) + 5(t + 3) \\ &= (2s + 5)(t + 3) \end{aligned}$$

4. Answers may vary, e.g.,
i) This trinomial has perfect squares for both its first coefficient and constant. Their respective square roots are $5x$ and 3. Also, $-(2 \times 5x \times 3) = -30x$, which is the other term. Therefore, this is a perfect-square trinomial with two equal factors of $(5x - 3)$.
$25x^2 - 30x + 9 = (5x - 3)^2$.
ii) Use an area diagram. Try different combinations of lengths and widths until you create a rectangle, such as this, with the desired area of $25x^2 - 30x - 9$.



The length and width are both $(5x - 3)$; therefore, the expression can be factored as
$25x^2 - 30x + 9 = (5x - 3)^2$.
b) Answers may vary, e.g., I prefer the perfect square method because it is more direct and does not need trial and error.
5. a) Determine two factors of -77 whose sum is 4.
The factors are 11 and -7.
Thus, $x^2 + 4x - 77 = (x + 11)(x - 7)$.
b) Determine two factors of -10 whose sum is -3.
The factors are -5 and 2.
Thus, $a^2 - 3a - 10 = (a - 5)(a + 2)$
c) First divide out a common factor of 3 and then determine two factors of 4 whose sum is -4. The factors are both -2.

$$\begin{aligned} 3x^2 - 12x + 12 &= 3(x^2 - 4x + 4) \\ &= 3(x - 2)^2 \end{aligned}$$

d) First divide out a common factor of m and then determine two factors of -4 whose sum is 3. The factors are -1 and 4.

$$\begin{aligned} m^3 + 3m^2 - 4m &= m(m^2 + 3m - 4) \\ &= m(m - 1)(m + 4) \end{aligned}$$

6. a) The first coefficient is 6, and the constant is -2. Determine two factors of $6 \times (-2) = -12$ that also sum to -1. The factors are -4 and 3. Thus:

$$\begin{aligned} 6x^2 - x - 2 &= 6x^2 + 3x - 4x - 2 \\ &= 3x(x + 2) - 2(2x + 1) \\ &= (3x - 2)(2x + 1) \end{aligned}$$

b) First divide out a common factor of 2. Now the first coefficient is 4 and the constant is -3. Determine two factors of $4 \times (-3) = -12$ whose sum is 4. The factors are -2 and 6.

$$\begin{aligned} 8n^2 + 8n - 6 &= 2(4n^2 + 4n - 3) \\ &= 2(4n^2 - 2n + 6n - 3) \\ &= 2[2n(2n - 1) + 3(2n - 1)] \\ &= 2(2n + 3)(2n - 1) \end{aligned}$$

c) Both the first coefficient and constant are perfect squares, with square roots of $3x$ and 2 respectively. Also, $2 \times 3x \times 2 = 12x$, which is the other term. Therefore, this is a perfect-square trinomial, with two equal factors of $(3x + 2)$.
Thus, $9x^2 + 12x + 4 = (3x + 2)^2$.
d) First divide out a common factor of a. Now the first coefficient is 6 and the constant is -4. Determine two factors of $6 \times (-4) = -24$ whose sum is 5. The factors are -3 and 8.

$$\begin{aligned} 6ax^2 + 5ax - 4a &= a(6x^2 + 5x - 4) \\ &= a(6x^2 - 3x + 8x - 4) \\ &= a[3x(2x - 1) + 4(2x - 1)] \\ &= a(3x + 4)(2x - 1) \end{aligned}$$

7. Answers may vary, e.g.,
a) Use an area diagram to express the area of the poster. Try different lengths and widths until you find a combination that gives the desired area.

	$2x$	3
x		$3x$
4	$8x$	12

Therefore, the poster could have a length of $2x + 3$ and a width of $x + 4$.
b) The new width is $2(x + 4) = 2x + 8$. The new length is $(2x + 3) + 2 = 2x + 5$. Therefore, the expression for the area becomes:

$$\begin{aligned} A &= (2x + 5)(2x + 8) \\ &= 4x^2 + 10x + 16x + 40 \\ &= 4x^2 + 26x + 40 \end{aligned}$$

c) First divide out a common factor of 9. From part **a)**, $(2x^2 + 11x + 12)$ has factors of $(2x + 3)(x + 4)$.

$$\begin{aligned} A &= 18x^2 + 99x + 108 \\ &= 9(2x^2 + 11x + 12) \\ &= 9(2x^2 + 11x + 12) \\ &= 9(2x + 3)(x + 4) \\ &= (6x + 9)(3x + 12) \end{aligned}$$

Thus, the poster could have a length of $6x + 9$ and a width of $3x + 12$.

8. a) Both the first coefficient and constant in this binomial are perfect squares separated by a negative sign, with square roots of $15x$ and 2 respectively. Therefore, this is a difference of squares.
$225x^2 - 4 = (15x - 2)(15x + 2)$
b) Both the first coefficient and constant in this trinomial are perfect squares, with square roots of 3 and 8 respectively. Also, $-(2 \times 3 \times 8) = -48$, which is the coefficient of the a term. Therefore, this is a perfect-square trinomial with two equal factors of $(3a - 8)$.
$9a^2 - 48a + 64 = (3a - 8)^2$
c) Both terms in this binomial are perfect squares, separated by a negative sign. Their square roots are x^3 and $2y$ respectively. Therefore, this is a difference of squares.
$x^6 - 4y^2 = (x^3 - 2y)(x^3 + 2y)$
d) Both the first term and constant in this trinomial are perfect squares, with square roots of $3 + n$ and 5. Also, $-[2 \times (3 + n) \times 5] = -10(3 + n)$, which is the other term. Therefore, this is a perfect-square trinomial and can be factored as $(3 + n - 5)^2$ or $(n - 2)^2$.
9. Answers may vary, e.g., begin by factoring the equation. By inspection, the equation factors to $y = (2x - 1)(x - 5)$. On inspection of the equation in factored form, this parabola has zeros at $x = 0.5$ and $x = 5$. By symmetry, the x-coordinate of the vertex exists halfway between the zeros, at $x = 2.75$. Substitute this value into the equation to solve for the y-value.

$$\begin{aligned} y &= (2x - 1)(x - 5) \\ &= [2(2.75) - 1][(2.75) - 5] \\ &= (5.5 - 1)(2.75 - 5) \\ &= (4.5)(-2.25) \\ &= -10.125 \end{aligned}$$

Therefore, the vertex occurs at (2.75, –10.125).

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Answers may vary, e.g.,
1. Quadratic Binomials:
Factorable: One type of quadratic binomial that is factorable is a difference of squares. A difference of squares takes the form $a^2 - b^2$, and factors to $(a - b)(a + b)$. For instance:
(1) $16x^2 - 49 = (4x - 7)(4x + 7)$
Not Factorable: Use the same values from before, but remove the negative sign from the constant. The expression is no longer a difference of squares, and thus is not factorable. (2) $16x^2 + 49$ = Not Factorable
Quadratic Trinomials:
Factorable: The simplest type of factorable trinomial factors to the form $(x + r)(x + s)$. Let $r = 4$ and $s = -7$. Then:

$$\begin{aligned} (x + r)(x + s) &= (x + 4)(x - 7) \\ &= x^2 + 4x - 7x - 28 \\ &= x^2 - 3x - 28 \end{aligned}$$

Therefore: (3) $x^2 - 3x - 28 = (x + 4)(x - 7)$

Another type of factorable trinomial factors to the form $a(px + r)(qx + s)$. Let $a = 1$, $p = 2$, $q = 5$, $r = 3$, $s = 2$, and $x = z$. Then:

$$\begin{aligned} a(px + r)(qx + s) &= (2z + 3)(5z + 2) \\ &= 10z^2 + 15z + 4z + 6 \\ &= 10z^2 + 19z + 6 \end{aligned}$$

Therefore: (4) $10z^2 + 19z + 6 = (2z + 3)(5z + 2)$
A third type of factorable trinomials are perfect-square trinomials. These trinomials can either take the form $a^2 - 2ab + b^2 = (a - b)^2$ or the form $a^2 + 2ab + b^2 = (a + b)^2$, and may be multiplied by a constant.
Choosing the first form, let $a = 4w$ and $b = 11$.
$a^2 - 2ab + b^2 = (a - b)^2$
$(4w)^2 - 2(4w)(11) + (11)^2 = ((4w) - (11))^2$
Therefore
(5) $16w^2 - 88w + 121 = (4w - 11)^2$
Choosing the second form, let $a = 5$, $b = 2p$, and multiply everything by a constant of –4. Therefore:
$-4(a^2 + 2ab + b^2) = -4(a + b)^2$
$-4((5)^2 + 2(5)(2p) + (2p)^2) = -4((5) + (2p))^2$
$-4(25 + 20p + 4p^2) = -4(5 + 2p)^2$
(6) $-100 - 80p - 16p^2 = -4(5 + 2p)^2$
Not Factorable: Consider the product of the first coefficient and constant. A trinomial is not factorable if it is impossible to add two factors of this product to get the coefficient of the middle term. That is, if the quadratic is in the form $ax^2 + bx + c$, then no factors of ac add to b.
Let $a = 3$ and $c = -5$. Then $ac = -15$, which has factors of $\pm 1, \pm 3, \pm 5, \pm 15$. It is impossible to add any two of these factors of opposite sign to –9 for instance. Let $b = -9$, and let $x = t$. Then the following is not factorable:
(7) $3t^2 - 9t - 5$ = Not Factorable
To generate another non-factorable trinomial that looks like a perfect-square trinomial, let $a = 4$ and $c = 9$ such that they are perfect squares and $ac = 36$. It is impossible to for any two numbers whose product is 36 to have a sum of 18 for instance. Let $b = 18$ and let $x = s$. Then the following is not factorable:
(8) $4s^2 + 18s + 9$ = Not Factorable
Four Terms:
Factorable: To create a situation that has four terms and is factorable, make the first three terms a perfect-square trinomial that when combined with the last term makes a difference of squares. The final result should be of the form $a^2 - b^2$, which factors to $(a - b)(a + b)$. Choose a to be a binomial so that a^2 expands to a perfect-square trinomial. If a takes the form $px + q$, then $a^2 = p^2x^2 + 2pqx + q^2$.
Let $a = (2x + 13)$ and $b = 9y$.
Factor $a^2 - b^2$.
$(2x + 13)^2 - 81y^2 = (2x + 13 - 9y)(2x + 13 + 9y)$
Expand $a^2 - b^2$.
$(2x + 13)^2 - 81y^2 = 4x^2 + 52x + 169 - 81y^2$
Therefore (9) $4x^2 + 52x + 169 - 81y^2$
$= (2x + 13 - 9y)(2x + 13 + 9y)$

To create a situation that has four terms and is not factorable, choose four terms that have no common factors and that cannot be grouped. For instance: (10)
$x^5 - x^3y^3 + x^2y + y^4$ = Not Factorable
This expression has no common factors, and a grouping method results in two unequal terms.
$x^5 - x^3y^3 + x^2y + y^4 = x^3(x^2 - y^3) + y(x^2 + y^3)$
= Not Factorable

2. Form a group and decide the order of play.

3. Read the instructions provided in the task to learn how to play.

4. & 5. Turn 1: Suppose you draw $3x^2 - 18x + 24$. You notice that 3 can be factored out of the expression.
$3x^2 - 18x + 24 = 3(x^2 - 6x + 8)$
You need two numbers that have a product of 8 and a sum of –6. You correctly choose the numbers –2 and –4. Thus, you correctly factor the polynomial to $3(x - 2)(x - 4)$, earning 5 points.

Turn 2: Suppose you draw $-8t^2 + 8t - 18$. You begin by factoring –2 out of the expression.
$-8t^2 + 8t - 18 = -2(4t^2 - 4t + 9)$
This looks like a perfect-square trinomial; so, you hastily write that it factors to $-2(2t - 3)^2$. Unfortunately, you have been tricked. The square roots of the first term and constant are $2t$ and 3 respectively, and $-(2 \times 2t \times 3) = -12t$, which is not equal to $-4t$. This polynomial is actually not fully factorable, so you earn no points.

Turn 3: Suppose you draw $10z^2 + 19z + 6$. Through an error in your area diagram, you conclude that this factors to $(z + 2)(10z + 3)$, when really it factors to $(2z + 3)(5z + 2)$, earning no points.

Turn 4: Suppose you draw $4x^2 + 52x + 169 - 81y^2$. You remember creating this polynomial using perfect-square trinomials and difference of squares. You factor correctly as follows, earning 5 points:
$4x^2 + 52x + 169 - 81y^2$
$= (2x + 13)^2 - 81y^2$
$= (2x + 13 - 9y)(2x + 13 + 9y)$

Turn 5: Suppose you draw $64c^2 - 64c + 16$. First, you first that 16 can be factored out of the equation.
$64c^2 - 64c + 16 = 16(4c^2 - 4c + 1)$
You observe that the trinomial in brackets has both a first coefficient and a constant that are perfect squares, with square roots of $2c$ and 1 respectively. Also, $-(2 \times 2c \times 1) = -4c$, which is the middle term. Therefore, this is a perfect-square trinomial with two equal factors of $(2c + 1)$. You correctly factor the given polynomial as $16(2c + 1)^2$, earning 5 points.

Turn 6: Suppose you draw $4s^2 - 10s - 121$. At first, this looks like a perfect-square trinomial, since both the first term and constant appear to be perfect squares. Then, you notice the negative sign in front of the constant, which means it is not a perfect square. Also, you cannot determine any factors of
$4 \times (-121) = -484$ that add to –10. Therefore, you correctly conclude that this polynomial is not factorable.
You think that you can make this polynomial factorable if you change it to a perfect-square trinomial. First, you change the constant from –121 to 121 to make it a perfect square. You want a perfect-square trinomial of the form $a^2 - 2ab + b^2$. Let $a = 4s$ and $b = 11$. Thus, the middle term becomes
$-(2 \times 4s \times 11) = -88s$. You make two changes to the polynomial, changing it to $4s^2 - 88s + 121$, which factors to $(2x - 11)^2$. The changes earn you two points.

Turn 7: Suppose you draw $3w^2 - 11w - 9$. The product of the first coefficient and the constant is
$3 \times (-9) = -27$. You try to find two factors of –27 that add to –11, but you cannot. Therefore, you correctly conclude that this polynomial is not factorable. You decide to try to change the middle term to make it factorable. You know that if you take two factors of
–27 and add them, you will determine a possible coefficient of w. Unfortunately, you choose the factors 1 and 27, whose product is 27 rather than –27. You generate the polynomial $3w^2 + 28w - 9$, which is not factorable, earning no points.

Turn 8: Suppose you draw $-8p^2 + 38p - 9$. The first coefficient is –8, and the constant is –9. You decide that you need to determine two numbers with a product of
$-8 \times (-9) = 72$ and a sum of 38.
Unfortunately, you decide that no such numbers exist, whereas 2 and 36 would give the correct result. Therefore, you determine that the polynomial is not factorable, when it can actually be factored to
$-(2p - 9)(4p - 1)$, earning no points.

Turn 9: Suppose you draw $12y^2 + 33y + 18$. First, you remove a common factor of 3 from each term.
$12y^2 + 33y + 18 = 3(4y^2 + 11y + 6)$
Then draw an area diagram to factor the trinomial in brackets.

	$4y$	3
y	$4y^2$	$3y$
2	$8y$	6

The diagram has a length of $(4y + 3)$ and a width of $(y + 2)$. Therefore, you conclude this trinomial factors to $3(y + 2)(4y + 3)$, earning 5 points.

Turn 10: Suppose you draw $16x^2 + 49$. Both terms of this binomial are perfect squares, which makes you think that this might be a difference of squares situation. However, both terms are positive, so this is incorrect. The coefficients of the two terms multiply to 784, but since this number is positive, there are no pairs of factors that will add to zero. Therefore, you conclude that this polynomial is not factorable. You notice that if you change the positive sign to a negative sign, this binomial becomes a difference of squares with $a = 4x$ and $b = 7$. Thus:
$16x^2 - 49 = (4x - 7)(4x + 7)$
This one change earns you 1 point.
You make a chart of all the polynomials you drew, their correct factors, and the points you earned for each.

Polynomial	Factors	Points Earned
$3x^2 - 18x + 24$	$3(x - 2)(x - 4)$	5
$-8t^2 + 8t - 1$	Not Factorable	0
$10z^2 + 19z + 6$	$(2z + 3)(5z + 2)$	0
$4x^2 + 52x + 169 - 81y^2$	$(2x + 13 - 9y)$ $(2x + 13 + 9y)$	5
$64c^2 - 64c + 16$	$16(2c - 1)^2$	5
$4s^2 - 10s - 121$	Not Factorable	2
$3w^2 - 11w - 9$	Not Factorable	0
$-8p^2 + 38p - 9$	$-(2p - 9)(4p - 1)$	0
$12y^2 + 33y + 18$	$3(y + 2)(4y + 3)$	5
$16x^2 + 49$	Not Factorable	1

5. After 10 turns, you add your scores. You scored a total of 23 points, putting you into second place.
6. You discuss polynomials that are difficult to factor with the class. The class agrees that polynomials with larger coefficients are more difficult to factor, such as $10z^2 + 19z + 6$. Also, you mention the difficulty of factoring polynomials with 4 terms when they involve both a perfect-square trinomial and a difference of squares, such as in your polynomial $4x^2 + 52x + 169 - 81y^2$. Finally, some students agree that polynomials that look like perfect-square trinomials but are not can be tricky, such as the polynomial $4s^2 - 10s - 121$.